

MATH 251: Quiz 4

October 22, 2015

Name: Solutions Sec: \_\_\_\_\_

1. Use linear (tangent plane) approximations to approximate the value of

$$((5.01)^2 + (9.98)^2)^{1/3}.$$

You need to pick a function  $f$  and do the linear approximation. Hint:  $5^3 = 125$ .

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$$\text{Tangent Plane: } z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

$$f(x, y) = (x^2 + y^2)^{1/3} \quad (x, y) = (5, 10)$$

$$f(5, 10) = (25 + 100)^{1/3} = 5.$$

$$f_x(x, y) = \frac{1}{3} \cdot 2x (x^2 + y^2)^{-2/3}$$

$$f_x(5, 10) = \frac{2}{3} \cdot 5 (125)^{-2/3} = \frac{2}{15} \quad (2)$$

$$f_y(x, y) = \frac{1}{3} \cdot 2y (x^2 + y^2)^{-2/3}$$

$$f_y(5, 10) = \frac{2}{3} \cdot 10 \cdot (125)^{-2/3} = \frac{20}{75} = \frac{4}{15}$$

Thus

$$f(5.01, 9.98) \approx 5 + \frac{2}{15}(5.01 - 5) + \frac{4}{15}(9.98 - 10)$$

$$= 5 + \frac{.02}{15} - \frac{.08}{15} = 5 - \frac{.06}{15} = 5 - \frac{.02}{5}$$

$$= 5 - .004 \quad (2)$$

$$= \boxed{4.996}$$

2. Find and characterize all critical points of

$$f(x, y) = x^3 + 6xy + 3y^2$$

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$$f_x = 3x^2 + 6y \Rightarrow 3x^2 - 6x = 0 \Rightarrow 3x(x-2) = 0$$

$$f_y = 6x + 6y = 0 \Rightarrow x = -y \quad (1)$$

Critical points:  $(0, 0) + (2, -2)$

$$f_{xx} = 6x \quad f_{xy} = 6 \Rightarrow D = 36x - 36$$

$$f_{yy} = 6$$

$(0, 0)$ ,  $D = -36$   
 $\Rightarrow$  saddle pt.

$(2, -2)$   $D = 36 > 0$

$f_{yy} > 0$  (2)

$\Rightarrow$  minimum

3. If  $f(x, y) = x^2 + xy$  where  $x$  and  $y$  are written in terms of  $s$  and  $t$  as

$$3x^2 + 4xy + 5y^2 \quad x(s, t) = 3s + t \quad y(s, t) = 2s + 5t^2$$

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compute  $\frac{\partial f}{\partial s}$ .

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial s}$$

$$= (2xy + y^2)(6s) + (x^2 + 2xy)(2)$$

$$= 12s(3s^2 + t)(2s + 5t^2)^2$$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial s} \quad (1)$$

$$= (6x + 4y)(3) + (4x + 10y)(2)$$

$$= 26x + 32y$$

$$= 26(3s + t) + 32(2s + 5t^2)$$

(1)