MATH 251: Quiz 3

October 8, 2015

Name: Solutions Sec: ____________

1. Let $\vec{r}(t) = \langle 4t, \cos(3t), \sin(3t)\rangle$.

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(a) Find the length of $\vec{r}(t)$ between $t = 0$ and $t = 2$.

(b) Compute the curvature of $\vec{r}$ at $t = 1$.

Curvature Formulas:

$$\kappa(t) = \left|\left|\frac{d\vec{T}}{ds}\right|\right| \qquad \kappa(t) = \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3} \qquad \kappa(x) = \frac{|f''(x)|}{(1 + f'(x)^2)^{3/2}}$$

a) $\vec{r}'(t) = \langle 4, -3\sin(3t), 3\cos(3t)\rangle$ (1)

$||\vec{r}'(t)|| = \sqrt{16 + 9\cos^2 3t + 9\sin^2(3t)} = \sqrt{16+9} = 5.$

So $\text{Length} = \int_0^2 ||\vec{r}'(t)||\,dt = \int_0^2 5\,dt = \boxed{10}$ (1)

(b) $\vec{r}''(t) = \langle 0, -9\cos(3t), -9\sin(3t)\rangle$.

$\vec{r}'(t) \times \vec{r}''(t) = \langle 27\sin^2(3t) + 27\cos^2(3t), 36\sin(3t), -36\cos(3t)\rangle$

$= \langle 27, 36\sin(3t), -36\cos(3t)\rangle.$ (1)

$||\vec{r}'(t) \times \vec{r}''(t)|| = \sqrt{27^2 + 36^2\sin^2(3t) + 36^2\cos^2(3t)} = \sqrt{27^2+36^2}$

$= 9\sqrt{3^2+4^2} = 45$

$\Rightarrow \kappa(t) = \dfrac{45}{(5)^3} = \dfrac{9}{25}$ (1)

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2. Compute f_{xx}, f_{xy}, and f_{yy} for $f(x,y) = x^3 + 3x^2y + 4y^2\sin(x)$.

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$f_x = 3x^2 + 6xy + 4y^2\cos(x)$

$f_{xx} = 6x + 6y - 4y^2\sin(x)$ (1)

$f_{xy} = 6x + 8y\cos(x)$ (1)

$f_y = 3x^2 + 8y\sin(x)$

$f_{yy} = 8\sin(x)$ (1)

3. Explain why

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$$g(x,y) = \frac{x^2 + ye^{x^2}}{x^2 + y^2 + 1}$$

is continuous at $(x,y) = (1,2)$. [Hint: Use the form of this function, and that functions you know from Calculus I are continuous.] Use this to compute

$$\lim_{(x,y)\to(1,2)} g(x,y).$$

(1)

Top and bottom are continuous, and the bottom is not zero. Therefore.

$$\lim_{(x,y)\to(1,2)} g(x,y) = g(1,2) \quad (1)$$

$$= \frac{1+2e^1}{1+4+1} = \boxed{\frac{1+2e}{6}} \quad (1)$$