MATH 251: Quiz 2

September 24, 2015

Name: Solutions Sec: ____________

1. Given the vectors $\vec{v} = \langle -1, 2, 1\rangle$ and $\vec{w} = \langle 0, 2, 3\rangle$.

(a) Write the equations of the lines $\vec{r}_1(t)$ and $\vec{r}_2(t)$ through the point $(2, 3, 1)$ with direction vectors $\vec{v}$ and $\vec{w}$ respectively.

(b) Compute $\vec{v} \times \vec{w}$.

(c) Find the equation (any form) of the plane through $(2, 3, 1)$ containing the vectors $\vec{v}$ and $\vec{w}$.

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(a)

$$\vec{r}_1(t) = \langle 2,3,1\rangle + t\langle -1,2,1\rangle$$
$$\vec{r}_2(t) = \langle 2,3,1\rangle + t\langle 0,2,3\rangle.$$

(1)

(b)

$$\vec{v} \times \vec{w} = \det \begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ 0 & 2 & 3 \end{vmatrix}$$

(1)

$$= (6-2)i - (-3-0)j + (-2-0)k$$
$$= \langle 4, 3, -2\rangle.$$

(1)

(c)

$$\begin{cases} 4(x-2) + 3(y-3) - 2(z-1) = 0 \\ 4x + 3y - 2z = 15 \end{cases}$$

(2)

2. For $\vec{r}(t) = \langle t^2 - 1 + e^t, \sin(2t) + 4, e^{t^2} + t^3 - t\rangle$, compute

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(a) $\lim_{t\to 1} \vec{r}(t)$.

(b) $\vec{r}'(t)$ as a function of t.

Everything is done componentwise:

a) $\lim_{t\to 1} \vec{r}(t) = \langle e, \sin(2)+4, e\rangle.$ (1)

b) $\vec{r}'(t) = \langle 2t+e^t, 2\cos(2t), 2te^{t^2}+3t^2-1\rangle.$ (1)

3. Parametrize the intersection of the cylinder $y^2 + z^2 = 9$ with the surface $3x + 4y^2 - z^2 = 4$.

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$y = 3\cos t$

$z = 3\sin t$ (1)

$0 \le t \le 2\pi$

$y = t$

$z = \pm\sqrt{9-t^2}$

$3x + 4t^2 - (9 - t^2) = 4$

(1) $x = \frac{4}{3} - \frac{5}{3}t^2 + 3$

$3x + 4(9\cos^2 t) - 9\sin^2 t = 4$

$3x = 4 + 9\sin^2 t - 36\cos^2 t$

$x = \frac{4}{3} + 3\sin^2 t - 12\cos^2 t.$

$$\boxed{\vec{r}(t) = \left\langle \frac{4}{3} + 3\sin^2 t - 12\cos^2 t, 3\cos t, 3\sin t\right\rangle \quad 0 \le t \le 2\pi}$$ (1)