Recitation 1: Extra Practice

September 2, 2014

1. Factor these quadratic expressions.

a) $x^2 - 25$

$(x+5)(x-5)$

b) $x^2 + 6x + 9$

$(x+3)(x+3) = (x+3)^2$

c) $x^2 - 8x + 15$

$(x-5)(x-3)$

d) $3x^2 + 11x - 4$

$(3x-1)(x+4)$

e) $x^2 + x - 21$

Quadratic Formula

$x = \frac{-1 \pm \sqrt{1+4 \cdot 1 \cdot 21}}{2} = \frac{-1 \pm \sqrt{85}}{2}$

So, this factors as

$\left(x - \frac{-1+\sqrt{85}}{2}\right)\left(x - \frac{-1-\sqrt{85}}{2}\right)$

f) $2x^2 + 4x - 1$ - Quadratic formula

$x = \frac{-4 \pm \sqrt{16+4 \cdot 1 \cdot 2}}{2 \cdot 2} = \frac{-4 \pm \sqrt{24}}{4}$

$= \frac{-4 \pm 2\sqrt{6}}{4} = \frac{-2 \pm \sqrt{6}}{2}$

So, we have

$\left(x - \frac{-2+\sqrt{6}}{2}\right)\left(x - \frac{-2-\sqrt{6}}{2}\right)$

2. Simplify the following rational and radical expressions.

a) $\sqrt{450}$

$= \sqrt{45 \cdot 10} = \sqrt{5 \cdot 9 \cdot 5 \cdot 2}$

$= 5\sqrt{18}$

b) $\frac{225}{600}$

$= \frac{3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 2 \cdot 3 \cdot 2 \cdot 3 \cdot 2} = \frac{3}{8}$

c) $\sqrt{28(x-4)^2(x-5)}$

$= \sqrt{7 \cdot 4 \cdot (x-4)^2 (x-5)}$

$= 2(x-4)\sqrt{7(x-5)}$

d) $\frac{x^2-4x+4}{x^2+x-6}$

$= \frac{(x-2)(x-2)}{(x-2)(x+3)} = \frac{x-2}{x+3}$

e) $\sqrt{72(x^2-4)(x^2+5x+6)}$

$= \sqrt{36 \cdot 2 (x+2)(x-2)(x+2)(x+3)}$

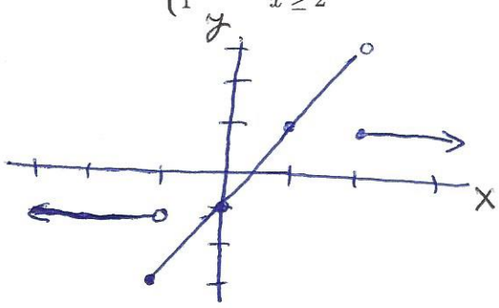
$= 6(x+2)\sqrt{2(x-2)(x+3)}$

f) $\frac{x^2-9}{x^2+6x+9}$

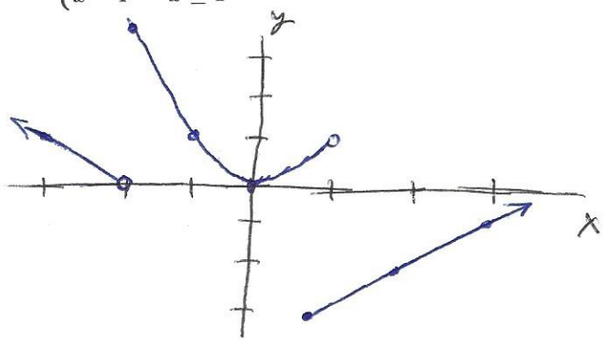
$= \frac{(x+3)(x-3)}{(x+3)(x+3)} = \frac{x-3}{x+3}$

3. Graph the following piecewise-defined functions.

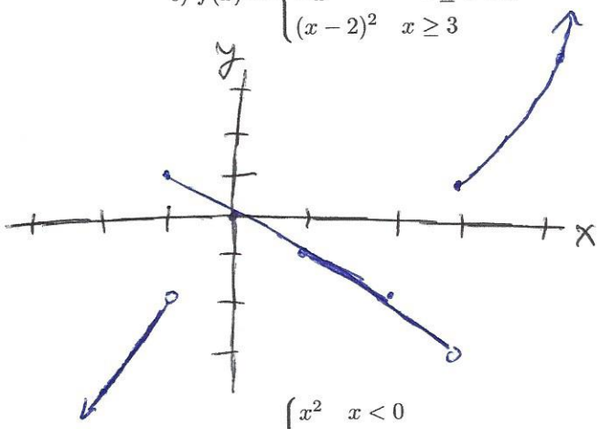
a) $f(x) = \begin{cases} -1 & x < -1 \\ 2x - 1 & -1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$



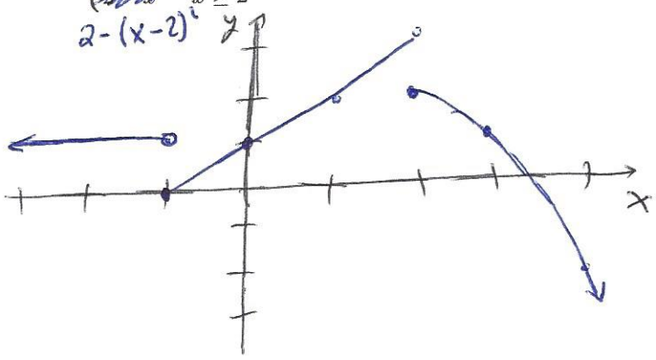
b) $f(x) = \begin{cases} -x - 2 & x < -2 \\ x^2 & -2 \le x < 1 \\ x - 4 & x \ge 1 \end{cases}$



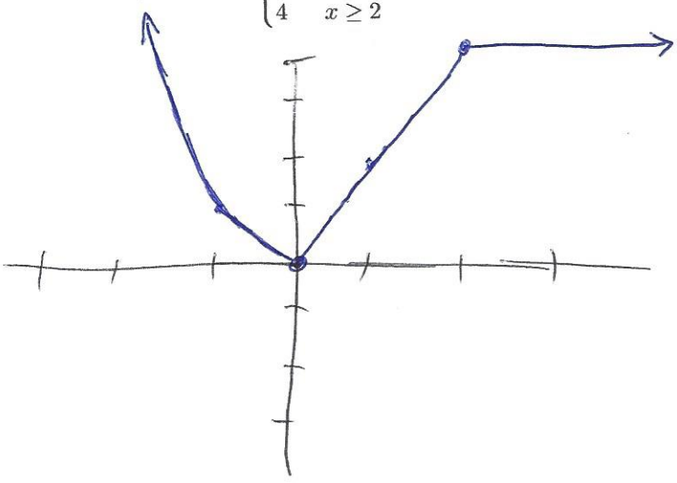
c) $f(x) = \begin{cases} 2x & x < -1 \\ -x & -1 \le x < 3 \\ (x-2)^2 & x \ge 3 \end{cases}$



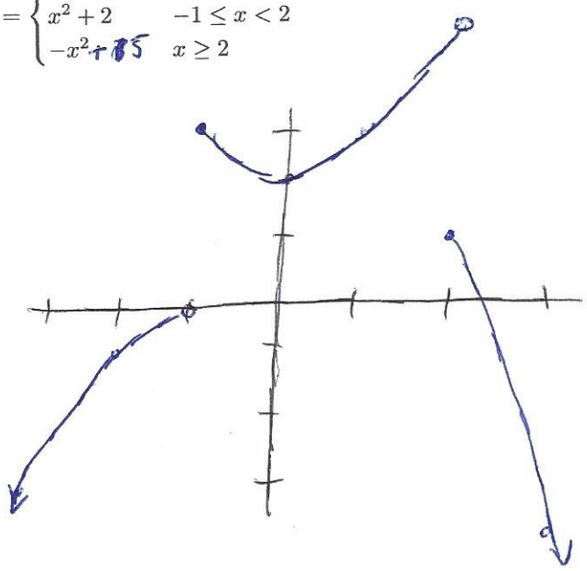
d) $f(x) = \begin{cases} 1 & x < -1 \\ x + 1 & -1 \le x < 2 \\ 2 - (x-2)^2 & x \ge 2 \end{cases}$



e) $f(x) = \begin{cases} x^2 & x < 0 \\ 2x & 0 \le x < 2 \\ 4 & x \ge 2 \end{cases}$



f) $f(x) = \begin{cases} -(x+1)^2 & x < -1 \\ x^2 + 2 & -1 \le x < 2 \\ -x^2 + 15 & x \ge 2 \end{cases}$



4. Identify the following functions as even, odd, or neither.

a) $f(x) = x^4 + 2$

$f(-x) = (-x)^4 + 2 = x^4 + 2$

$\Rightarrow$ Even

b) $f(x) = x^2 + 2x$

$f(-x) = (-x)^2 + 2(-x) = x^2 - 2x$

$\Rightarrow$ Neither

c) $f(x) = x^5 + 4x^3 - x$

$f(-x) = (-x)^5 + 4(-x)^3 - (-x)$

$= -x^5 - 4x^3 + x = -f(x)$

$\Rightarrow$ ODD

d) $f(x) = \sin(x)$

$\sin(-x) = -\sin(x)$ $\Rightarrow$ Odd

e) $f(x) = \cos(x)$

$\cos(-x) = \cos(x)$

$\Rightarrow$ Even

f) $f(x) = |x|$

$|-x| = |x|$

$\Rightarrow$ Even

5. Identify the center and radius of the given circles.

a) $(x-2)^2 + (y+5)^2 = 16$

Center: $(2, -5)$

Radius $= 4$

b) $x^2 + y^2 + 4y + 4 = 25$

$x^2 + (y+2)^2 = 25$

Center: $(0, -2)$

Radius: 5

c) $x^2 - 6x + y^2 + 2x - 39 = 0$

$x^2 - 6x + 9 + y^2 + 2x + 1 = 39 + 9 + 1$

$(x-3)^2 + (y+1)^2 = 49$

Center: $(3, -1)$

Radius: 7

d) $x^2 + 4x + y^2 = 0$

$x^2 + 4x + 4 + y^2 = 4$

$(x+2)^2 + y^2 = 4$

Center: $(-2, 0)$

Radius: 2

6. For each pair, P, Q, give the distance between P and Q, as well as the midpoint of the segment $\overline{PQ}$.

a) $P = (0, 3)$, $Q = (1, -4)$.

$d = \sqrt{(1-0)^2 + (-4-3)^2} = \sqrt{1+49}$

$d = \sqrt{50}$

$\text{mid} = \left(\frac{0+1}{2}, \frac{3-4}{2}\right) = \left(\frac{1}{2}, -\frac{1}{2}\right)$

b) $P = (2, 5)$, $Q = (10, 13)$.

$d = \sqrt{8^2 + 8^2} = \sqrt{128} = 8\sqrt{2}$

midpt $= (6, 9)$

c) $P = (1, 4)$, $Q = (10, 4)$.

$d = \sqrt{9^2 + 0^2} = 9$

midpt $= (5\frac{1}{2}, 4)$

d) $P = (0, 0)$, $Q = (3, 4)$.

$d = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

midpt $= (3/2, 2)$