MATH 135: Quiz 8

October 28, 2014

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Name: Solutions Sec: ____________

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1. Use differentials or linear approximations to approximate the value of $\sqrt{16.1}$.

$f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2}x^{-1/2}$ (1)

$f(16.1) \approx f(16) + f'(16)(\Delta x)$ (1)

$\approx 4 + \frac{1}{2 \cdot 4}(.1)$

$\approx 4 + \frac{.1}{8} = \underline{4.0125}$ (1)

$df = f'(x_0)\,dx$ (1)

$= \frac{1}{2} \cdot \frac{1}{\sqrt{16}} \cdot .1 = \frac{.1}{8} = 0.0125$

$df \approx \Delta f = f(16.1) - f(16)$

$f(16.1) \approx df + f(16) = 4 + 0.0125 = \underline{\underline{4.0125}}$ (1)

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2. The Kinetic Energy of an object [in Joules] is given by the formula

$$KE(v) = \frac{1}{2}mv^2$$

where v is the velocity of the ball [in m/s] and m is the mass [in kg]. Someone throws a 1 kg medicine ball at the wall and you measure the speed of it as 10 m/s. [The mass is exactly 1 kg, so there is no error in that measurement.] If your measurement of the speed is within .5 m/s, what is the approximate propagated error (ΔKE) in your calculation of the Kinetic Energy of the ball?

$\Delta KE \approx dKE = KE'(v_0)\,\Delta v_0$ (1)

$KE'(v) = \frac{d}{dv}\left(\frac{1}{2}mv^2\right) = mv$

So $KE'(v_0) = (1\,kg)(10\,m/s) = 10\,kg\,m/s$

Thus $\Delta KE \approx (10\,kg\,m/s)(\pm .5\,m/s) = \pm 5\,kg\,m^2/s^2 = \boxed{\pm 5\,J}$ (1)

3. Let

$$f(x) = 2x^3 - 9x^2 = x^2(2x - 9).$$

(a) Find all "critical numbers" or "critical points" of f.

(b) Find the **absolute** maximum and minimum values of f on $[-1, 5]$.

(c) What does the Mean Value Theorem say about some point c between -1 and 5?

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(a) $f'(x) = 6x^2 - 18x = 6x(x-3)$ (1)

Critical numbers : $f'(x) = 0$ or $f'(x)$ does not exist.

- $f'(x)$ exists everywhere (1)
- $f'(x) = 0 \Rightarrow 6x(x-3) = 0 \Rightarrow \boxed{x = 0,\ x = 3}$

(b) Need to check critical numbers and endpoints.

$f(-1) = (-1)^2(2(-1)-9) = 1\cdot(-2-9) = -11$ (1)

$f(0) = 0^2(2(0)-9) = 0$

$f(3) = (3)^2(2(3)-9) = 9\cdot(6-9) = \boxed{-27 \leftarrow \text{Minimum}}$

$f(5) = (5)^2(2\cdot 5 - 9) = 25\cdot 1 = \boxed{25 \leftarrow \text{Maximum}}$ (1)

So Absolute min is -27 at $x=3$
Absolute max is 25 at $x=5$.

(c) MVT says that there is a c between -1 and 5 with

$6c(c-3) = \boxed{f'(c) = \frac{f(5) - f(-1)}{5 - -1} = \frac{25 - -11}{6} = \frac{36}{6} = 6.}$ (1)