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MATH 135: Quiz 7

October 21, 2014

Name: Solutions Sec: ______

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1. Use implicit differentiation to find $\frac{dy}{dx}$ for the equation below. Your answer can be left as a function of both x and y.

$$y^3 + x^2y^2 = 2x^2 + 2y + 4$$

$$\frac{d}{dx}\left[y^3 + x^2y^2\right] = \frac{d}{dx}\left[2x^2 + 2y + 4\right] \quad (1)$$

$$3y^2\frac{dy}{dx} + y^2 \cdot 2x + x^2 \cdot 2y\frac{dy}{dx} = 4x + 2\frac{dy}{dx} + 0 \quad (1)$$

$$\left(3y^2 + 2x^2y - 2\right)\frac{dy}{dx} = 4x - 2xy^2$$

$$\boxed{\frac{dy}{dx} = \frac{4x - 2xy^2}{3y^2 + 2x^2y - 2}} \quad (1)$$

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2. Find $\frac{dy}{dx}$ for the function below. Leave your answer **only** as a function of x. (Hint: Logarithmic differentiation)

$$y = x^{\sin x}$$

$$\ln y = \ln\left(x^{\sin x}\right) \quad (1)$$

$$\ln y = (\sin(x))(\ln(x))$$

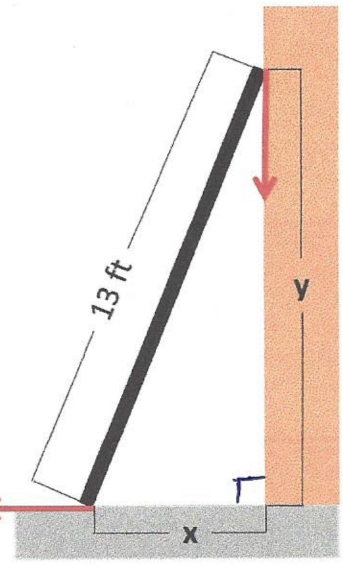
$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[\sin(x) \cdot \ln(x)] \quad (1)$$

$$\frac{1}{y}\frac{dy}{dx} = \cos(x)\ln(x) + \frac{\sin(x)}{x}$$

$$\frac{dy}{dx} = \left[\cos(x)\ln(x) + \frac{\sin(x)}{x}\right]y$$

$$\boxed{\frac{dy}{dx} = \left[\cos(x)\ln(x) + \frac{\sin(x)}{x}\right]x^{\sin(x)}} \quad (1)$$

3. A 13 ft long ladder is leaned up against a wall as shown in the picture on the right. As the bottom of the ladder slides away from the wall, the top slides down the wall. Let x represent the distance the base of the ladder is from the wall, and y, the height of the top of the ladder off the ground.



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(a) What is an equation relating the distances x and y in the figure? (Hint: The ground, wall, and ladder make up a right triangle)

(b) How high is the top of the ladder off the ground when the base is 5 feet from the wall?

(c) Differentiate your equation in (a) to get a relation between the changes of x and y with respect to time t.

(d) If the base of the ladder is sliding away from the wall at a rate of $\frac{dx}{dt} = 2$ ft/s when the base is 5 feet from the wall, how fast is the top sliding down at this moment?

(a) $\boxed{x^2 + y^2 = 13^2 = 169}$ (1)

(b) When $x = 5$, then

$$5^2 + y^2 = 169 \Rightarrow 25 + y^2 = 169 \Rightarrow y^2 = 144 \Rightarrow \boxed{y = 12\text{ ft}}$$ (1)

(c) $$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}\left[169\right]$$

$$\boxed{2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0}$$ (1)

(d) Plugging in $x = 5$ ft, $\frac{dx}{dt} = 2$ ft/s, and $y = 12$ ft from (b)

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(5)(2) + 2(12)\frac{dy}{dt} = 0$$

$$24\frac{dy}{dt} = -20$$

$$\boxed{\frac{dy}{dt} = \frac{-20}{24} = -5/6 \text{ ft/s}}$$ (1)