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MATH 135: Quiz 6

October 14, 2014

Name: Key Sec: _____

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1. Consider the table below defining values for the functions u and v and their derivatives.

x	1	2	3	4
$u(x)$	3	5	4	-1
$v(x)$	6	3	2	-3
$u'(x)$	4	-2	-3	1
$v'(x)$	-5	-1	2	6

Define the functions

$$f(x) = u(x)v(x) \qquad g(x) = v(u(x)) \qquad h(x) = \frac{v(x)}{u(x)+1}.$$

Compute the following derivatives using the numbers in the table above (and the rules you know for taking derivatives):

(a) $f'(4)$

(b) $g'(1)$

(c) $h'(3)$

(1)

(a) $f'(x) = u'(x)v(x) + u(x)v'(x)$, so (1)

$f'(4) = u'(4)v(4) + u(4)v'(4) = 4(1)\cdot 3 + (-1)\cdot 6 = \underline{\underline{-9}}$

(1)

(b) $g'(x) = v'(u(x))\cdot u'(x)$ so

$g'(1) = v'(u(1))\,u'(1) = v'(3)u'(1) = 2\cdot 4 = \underline{\underline{8}}$ (1)

(1)

(c) $h'(x) = \dfrac{(u(x)+1)v'(x) - v(x)u'(x)}{(u(x)+1)^2}$ so

$h'(3) = \dfrac{(u(3)+1)v'(3) - v(3)u'(3)}{(u(3)+1)^2} = \dfrac{5\cdot 2 - 2\cdot(-3)}{5^2}$

$= \dfrac{10+6}{25} = \boxed{\dfrac{16}{25}}$ (1)

2. The equation for the height $h(t)$ a falling object under gravitational acceleration g, initial velocity v_0, and initial height h_0 is

$$h(t) = -\frac{1}{2}gt^2 + v_0t + h_0. \quad = -4t^2 + v_0t + h_0.$$

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An adventurer has found himself on a planet fairly similar to that of Earth. He is standing on top of a tall building and **drops** a baseball off the edge. After 3 seconds, the ball has traveled a quarter of the way down the building (that is, the height after 3 seconds is $\frac{3}{4}$ of what it was initially). The gravitational acceleration on this planet is $g = 8\ m/s^2$.

(a) What is the initial velocity of the ball? (This does not require any work)

(b) What is the height of the building?

(c) At what time does the ball hit the ground?

(d) What is the impact velocity of the ball? (Velocity when the ball hits the ground)

(a) Dropped $\Rightarrow$ $v_0 = 0$ m/s. (1)

(b) We know $h(3) = 3/4\, h_0$, so.

$3/4\, h_0 = h(3) = -4(3)^2 + 0 + h_0.$

$3/4\, h_0 = -4 \cdot 9 + h_0$

$1/4\, h_0 = 36 \quad \Rightarrow \quad h_0 = 144m$ (1)

So $h(t) = -4t^2 + 144$

(c) Want $h(t) = 0$

$0 = -4t^2 + 144 \Rightarrow 4t^2 = 144 \Rightarrow t^2 = 36$

$\Rightarrow t = 6$ sec (1)

(d) $v(t) = h'(t) = -8t + 0$

So $v(6) = -8 \cdot 6 = -48$ m/s (1)