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MATH 135: Quiz 5

October 7, 2014

Name: Solutions Sec: ______

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1. The equation for the height $h(t)$ a falling object under gravitational acceleration g, initial velocity v_0, and initial height h_0 is

$$h(t) = -\frac{1}{2}gt^2 + v_0 t + h_0.$$

An astronaut is standing on the edge of a cliff on Jupiter's moon, Io. He throws a rock straight up, and it reaches its maximum height 2 seconds after it is thrown. This maximum height is 36 meters above the **bottom** of the cliff. The gravitational acceleration on Io is $g = 2\ m/s^2$.

(a) What is the initial velocity of the rock?

(b) What is the height of the cliff?

(c) At what time does the rock hit the ground?

(d) What is the impact velocity of the rock? (Velocity when the rock hits the ground)

$h(t) = -t^2 + V_0 t + h_0$ $\qquad$ $h'(t) = -2t + V_0$

Problem Statement gives: $\quad h'(2) = 0 \qquad h(2) = 36 \quad (1)$

- $h'(2) = 0 \Rightarrow -2(2) + V_0 = 0 \Rightarrow -4 + V_0 = 0 \Rightarrow \boxed{V_0 = 4\ m/s}$ (1)

Initial velocity.

- $h(2) = 36 \Rightarrow -(2)^2 + V_0(2) + h_0 = 36$

$\Rightarrow -4 + 4\cdot 2 + h_0 = 36$

$\Rightarrow 4 + h_0 = 36 \Rightarrow \boxed{h_0 = 32\ m}$ ✓ (1) height of Cliff

So $h(t) = -t^2 + 4t + 32 = -(t^2 - 4t - 32)$

$= -(t-8)(t+4)$ (1)

(c) Hitting the grand: $h(t) = 0 = -(t-8)(t+4)$

So, the rock hits the grand at $\boxed{t = 8\ sec}$ (1)

(d) Impact velocity $= h'(8) = -2(8) + V_0 = -2(8) + 4$

$= -16 + 4 = \boxed{-12\ m/s}$ (1)

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2. Consider the function

$$f(x) = (x^2 + 3x - 4)^3$$

(a) Find $f'(x)$.

(b) Find all points x where the graph of f has a horizontal tangent line.

(a) By the Chain Rule

$f'(x) = 3(x^2+3x-4)^2 \cdot (x^2+3x-4)'$ (1)

$$\left.\begin{aligned} &= 3(x^2+3x-4)^2(2x+3) \\ &= 3(2x+3)\big((x+4)(x-1)\big)^2 \\ &= 3(2x+3)(x+4)^2(x-1)^2. \end{aligned}\right\} \quad (1)$$

(b) Horizontal Tangent Line $\Rightarrow f'(x) = 0$

$0 = 3(2x+3)(x+4)^2(x-1)^2$ (1)

$\Downarrow$

$2x+3=0$ or $x+4=0$ or $x-1=0$

So f has a horizontal tangent at

$\boxed{x = -3/2, -4, 1}$ (1)