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MATH 135: Quiz 4

September 30, 2014

Name: Solutions Sec: ____________

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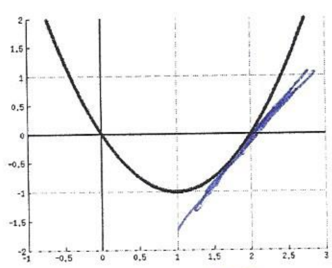
1. For each of the functions graphed below, answer whether or not the function is differentiable **at x=2**. If it is differentiable, then circle + (positive), - (negative), or 0 (zero) for the sign of the derivative **at x=2**. Sketching a tangent line may help.

(a)

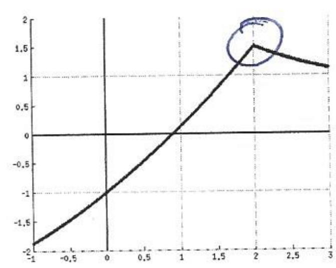


Differentiable: Yes / No

Sign of Derivative: + / - / 0

(1)

(b)

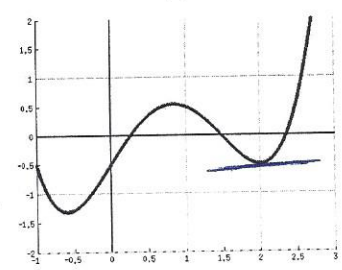


Differentiable: Yes / No

Sign of Derivative: + / - / 0

(1)

(c)



Differentiable: Yes / No

Sign of Derivative: + / - / 0

(1)

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2. Find the derivative of $f(x) = x^2 + 2x$ using the definition of derivative. Do not use any tricks for finding derivatives (power rule etc.).

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad (1)$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) - x^2 - 2x}{h} \quad (1)$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h - \cancel{x^2} - \cancel{2x}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \to 0} 2x + h + 2 = \boxed{2x + 2} \quad (1)$$

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3. Find the derivatives of the functions $f(x)$ and $g(x)$ below. You can use all derivative rules here. Please show all steps so I know what rules (product, quotient, etc.) that you are applying.

$$f(x) = e^x\left(\sin(x) - x^2\right) \qquad g(x) = \frac{x^5 + 3x^2 + 2}{x^{2/3}}$$

$$f'(x) = (e^x)'(\sin(x) - x^2) + e^x(\sin(x) - x^2)' \quad (1) \quad \text{[Prod]}$$

$$= e^x(\sin(x) - x^2) + e^x(\cos(x) - 2x).$$

$$\boxed{f'(x) = e^x(\sin(x) + \cos(x) - x^2 - 2x).} \quad (1)$$

$$g'(x) = \frac{(x^{2/3})(x^5 + 3x^2 + 2)' - (x^5 + 3x^2 + 2)(x^{2/3})'}{(x^{2/3})^2} \quad \text{[Quotient]} \quad (1)$$

$$= \frac{x^{2/3}(5x^4 + 6x) - (x^5 + 3x^2 + 2)(\tfrac{2}{3}x^{-1/3})}{x^{4/3}}$$

$$= \frac{1}{x^{4/3}}\left(5x^{14/3} + 6x^{5/3} - \tfrac{2}{3}x^{14/3} - 2x^{5/3} - \tfrac{4}{3}x^{-1/3}\right).$$

$$= \frac{1}{x^{4/3}}\left(\tfrac{13}{3}x^{14/3} + 4x^{5/3} - \tfrac{4}{3}x^{-1/3}\right).$$

$$\boxed{g'(x) = \tfrac{13}{3}x^{10/3} + 4x^{1/3} - \tfrac{4}{3}x^{-5/3}} \quad (1)$$

or $g(x) = x^{13/3} + 3x^{4/3} + 2x^{-2/3}.$ and by power rule

$$\boxed{g'(x) = \tfrac{13}{3}x^{10/3} + 4x^{1/3} - \tfrac{4}{3}x^{-5/3}} \quad \text{or } (2)$$