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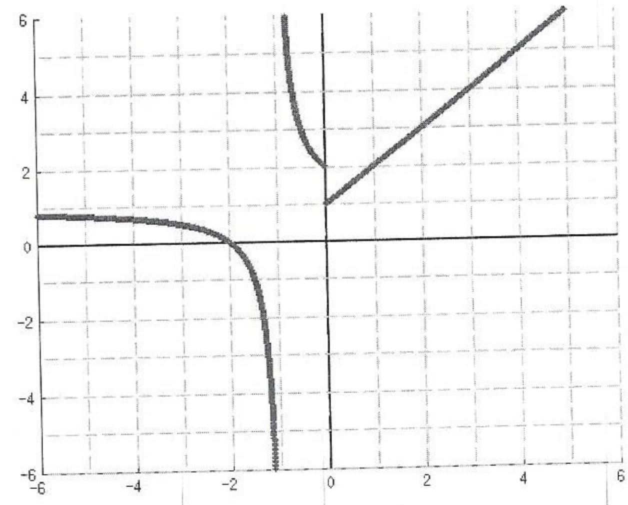
MATH 135: Quiz 3

September 23, 2014

Name: Solutions Sec: ______

1. Consider the piecewise-defined function f below.

$$f(x) = \begin{cases} \frac{x+2}{x+1} & x < 0 \\ x+1 & x \geq 0 \end{cases}$$



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(a) Find all points of discontinuity of f.

(b) Use the definition of continuity to show if f is continuous at $x = 0$. (You should be taking some limits here.)

(a) $x = -1$ [Pole] , $x = 0$ [Jump] (1)

(b) $f(0) = 1$ [$= x+1$]

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{x+2}{x+1} = \frac{\lim_{x \to 0^-} x+2}{\lim_{x \to 0^-} x+1} = \frac{0+2}{0+1} = \underline{2} \quad (1)$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x+1 = 0+1 = \underline{1} \quad (1)$$

Since $\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$, $\lim_{x \to 0} f(x)$ does not exist. (1)

Therefore, f is <u>not</u> continuous at $x=0$.

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2. Show that $g(x) = x^4 + 3x^3 - 10$ has a root ($g(x) = 0$) in $[-1, 2]$.

$g(-1) = (-1)^4 + 3(-1)^3 - 10 = 1 - 3 - 10 = -12 < 0$ (1)

$g(2) = 2^4 + 3(2)^3 - 10 = 16 + 24 - 10 = 30 > 0$

Since g is continuous, the Intermediate Value Theorem says that there is a c between -1 and 2 so that $g(c) = 0$. (1)

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3. The population of bacteria P in an ideal environment generally obeys the equation

$$P(t) = P_0 2^{kt} \qquad (1)$$

where P_0 and k are constants and t is in hours. Assume that we have a population of *E. coli* that follows the equation (1). This population doubles every 30 minutes, and at $t = 1$ hour $= 60$ minutes the population was 2,000.

(a) Find the constants P_0 and k.

(b) At what time t will the population reach 5,000? Express your answer as a logarithm.

(a) Doubling time $= .5$ hr implies $2P(0) = P(.5)$

$\Rightarrow 2 \cdot \cancel{P_0} 2^{k \cdot 0} = \cancel{P_0} 2^{k/2}$

$2^1 = 2^{k/2} \Rightarrow \log_2 2^1 = \log_2 2^{k/2}$

$1 = k/2 \quad \boxed{k = 2}$ (1)

Population data:

$2{,}000 = P(1) = P_0 2^{2 \cdot 1} = 4P_0$

So $\boxed{P_0 = 500}$ (1)

Equivalent answers accepted.

(b) $\dfrac{5{,}000}{500} = P(t) = \dfrac{500\, 2^{2t}}{500}$ (1)

$10 = 2^{2t} \Rightarrow \log_2 10 = 2t$

So $\boxed{t = \frac{1}{2} \log_2 10 = \log_2 \sqrt{10}}$ (1)