MATH 135: Quiz 2

September 16, 2014

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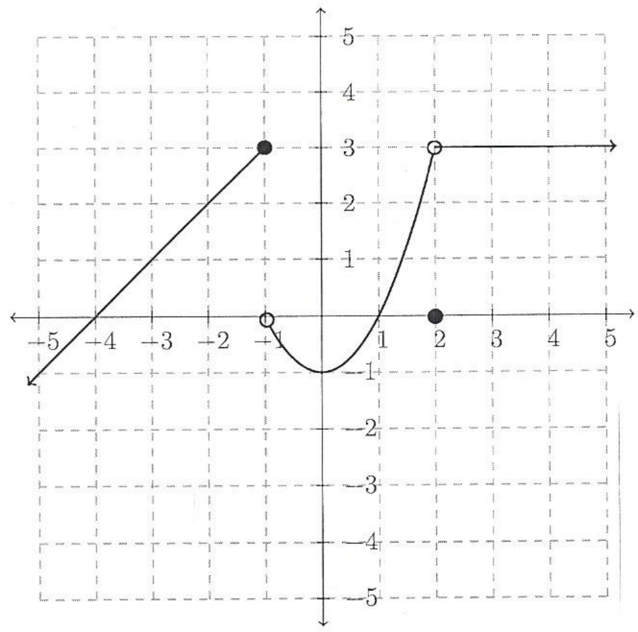
Name: Solutions Sec: ____________

1. Consider the piecewise-defined function f below.

$$f(x) = \begin{cases} x+4 & x \leq -1 \\ x^2-1 & -1 < x < 2 \\ 0 & x = 2 \\ 3 & x > 2 \end{cases}$$



/4 (a) Find $f(-1)$ and compute $\lim_{x\to -1} f(x)$ algebraically.

$f(-1) = (-1)+4 = 3.$ (1)

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (x+4)$

$= -1+4$

$\lim_{x\to -1^-} f(x) = 3$ (1)

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (x^2-1)$

$= (-1)^2 - 1$

$\lim_{x\to -1^+} f(x) = 0$ (1)

So $\lim_{x\to -1} f(x)$ does not exist because the left and right-hand limits are not the same. (1)

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(b) Find $f(2)$ and determine $\lim_{x\to 2} f(x)$ using the graph on the previous page.

$f(2)=0$ by definition. (1)

Looking at the graph, we see that the parabolic section just to the left of $x=2$ is approaching 3 as $x\to 2$.

Thus $\lim_{x\to 2^-} f(x) = 3.$ (1)

Similarly, the line/constant just to the right of $x=2$ is also approaching 3.

Thus $\lim_{x\to 2^+} f(x)=3$ (1)

Therefore, $\boxed{\lim_{x\to 2} f(x) = 3}$ (1)

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2. Evaluate the following limit: $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$

For $x\neq 4$, we can simplify,

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2} \quad (1)$$

$$= \lim_{x\to 4}\frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x}+2)}$$

$$= \lim_{x\to 4}\frac{1}{\sqrt{x}+2}$$

$$\boxed{\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \frac{1}{\sqrt{4}+2} = 1/4} \quad (1)$$

or

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4}\frac{\cancel{\sqrt{x}-2}}{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)} \quad (1)$$

$$= \lim_{x\to 4}\frac{1}{\sqrt{x}+2}$$

$$\boxed{\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}} \quad (1)$$