MATH 135: Quiz 13

December 9, 2014

Name: Solutions Sec: ____________

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1. Use the Fundamental Theorem of Calculus to compute the following:

$$\int_1^3 \frac{2x^3+3x^2+1}{x^2}\,dx$$

$$= \int_1^3 2x+3+\frac{1}{x^2}\,dx \quad (1)$$

$$= \left[x^2+3x-\frac{1}{x}\right]_1^3 \quad (1)$$

$$= 3^2+3\cdot 3-\frac{1}{3}-(1^2+3\cdot 1-1)$$

$$= 9+9-\frac{1}{3}-3 = \boxed{14\tfrac{2}{3}} = 44/3 \quad (1)$$

/2

$$\frac{d}{dx}\int_2^{x^2} (\cos(t))^{15}\,dt$$

$$= \frac{d}{du}\left[\int_2^u (\cos(t))^{15}\,dt\right]\frac{du}{dx} \qquad \underline{u=x^2}$$

$$= (\cos(u))^{15}\cdot 2x \quad (2)$$

$$\boxed{= 2x\,(\cos(x^2))^{15}}$$

$$x(x^2-2)^3\,dx = (x^2-2)^3\,(x\,dx)$$
$$= \frac{1}{2}(x^2-2)^3\,(2x\,dx)$$

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2. Find the area under the curve $y = x(x^2-2)^3$ between $x = 1$ and $x = 2$.

$$A = \int_1^2 x(x^2-2)^3\,dx \quad (1) \qquad u = x^2-2 \qquad u(1) = 1-2=-1$$
$$du = 2x\,dx \qquad u(2) = 4-2=2$$

$$[u.\text{sub}] = \int_{-1}^{2} \frac{1}{2}\cdot u^3\,du \quad (1)$$
$$= \frac{1}{2}\cdot \frac{u^4}{4}\Big|_{-1}^{2} = \frac{1}{2}\left(\frac{2^4}{4} - \frac{(-1)^4}{4}\right) \quad (1)$$
$$= \frac{1}{2}\left(\frac{16}{4} - \frac{1}{4}\right) = \boxed{15/8}$$

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3. Compute the following definite integral.

$$\int_0^{\sqrt[3]{\pi}} x^2 \sin(x^3)\;dx$$

u substitution:

$$u = x^3 \qquad u(0) = 0$$
$$du = 3x^2\,dx \qquad u(\sqrt[3]{\pi}) = \pi \quad (1)$$

$$\int_0^{\sqrt[3]{\pi}} x^2 \sin(x^3)\,dx = \int_0^{\sqrt[3]{\pi}} \sin(x^3)\,(x^2\,dx)$$
$$= \frac{1}{3}\int_0^{\sqrt[3]{\pi}} \sin(x^3)\,(3x^2\,dx)$$

So by u-sub

$$= \frac{1}{3}\int_0^{\pi} \sin(u)\,du$$
$$= \frac{1}{3}\left[-\cos(u)\right]_0^{\pi} = \frac{1}{3}\left[+(+1) + +(1)\right]$$
$$= \boxed{2/3} \quad (1)$$