MATH 135: Quiz 11

November 18, 2014

$\frac{}{10}$

Name: Solutions Sec: ____________

1. Find the vertical and horizontal asymptotes of the following functions

$$f(x) = \frac{e^{3x}+4}{e^{2x}-8}.$$

$\frac{}{3}$

$$\lim_{x\to\infty} \frac{\overset{\infty}{\overline{e^{3x}+4}}}{e^{2x}-8} = \frac{\infty}{\infty}$$

$$\text{L'Hôpitals} = \lim_{x\to\infty} \frac{3e^{3x}}{2e^{2x}} =$$

$$= \lim_{x\to\infty} \frac{3}{2} e^{x} = \infty$$

$\therefore$ No Asymptote (1)

$$\lim_{x\to-\infty} \frac{\overset{-\infty}{\overline{e^{3x}+4}}}{e^{2x}-8}$$

$$= \frac{0+4}{0-8} = \boxed{-1/2}$$

$$\boxed{y = -1/2}$$ (1)

Vertical

Denominator $= 0$

$$e^{2x} - 8 = 0$$

$$e^{2x} = 8$$

$$2x = \ln 8$$

$$\boxed{x = \frac{1}{2}\ln 8}$$ (1)

$$g(x) = \frac{e^{4x}-5}{e^{4x}+2}$$

$\frac{}{3}$

$$\lim_{x\to\infty} \frac{\overset{\infty}{\overline{e^{4x}-5}}}{e^{4x}+2} = \frac{\infty}{\infty}$$

$$\text{L'Hôp} = \lim_{x\to\infty} \frac{4e^{4x}}{4e^{4x}}$$

$$= \lim_{x\to\infty} 1 = 1$$

$$\boxed{y = 1}$$ (1)

$$\lim_{x\to-\infty} \frac{\overset{-\infty}{\overline{e^{4x}-5}}}{e^{4x}+2}$$

$$= \frac{0-5}{0+2} = -5/2$$

$$\boxed{y = -5/2}$$ (1)

Vertical

Denominator $= 0$

$$e^{4x} + 2 = 0$$

$$e^{4x} = -2$$

Never true!

$\therefore$ No vertical Asymptote. (1)

2.

1/2

(a) Use implicit differentiation to find $\frac{dy}{dx}$ if x and y satisfy the equation

$$\frac{d}{dx}\left(x^2y + 3y^2 = -2x^2 + 4y + 8\right)$$

$$2x\cdot y + x^2\frac{dy}{dx} + 6y\frac{dy}{dx} = -4x + 4\frac{dy}{dx} \quad (1)$$

$$(x^2+6y-4)\frac{dy}{dx} = -4x - 2xy$$

$$\boxed{\frac{dy}{dx} = \frac{-4x-2xy}{x^2+6y-4} = \frac{4x+2xy}{4-x^2-6y}} \quad (1)$$

1/2

(b) Use part (a) to write the equation for the tangent line to the graph of the equation above at the point $(1, 2)$

$$y - y_1 = m(x - x_1)$$

$$m = \left.\frac{dy}{dx}\right|_{(1,2)} = \frac{4(1) + 2(1)(2)}{4 - (1)^2 - 6(2)} = \frac{4+4}{4-1-12} = \frac{8}{-9} \quad (1)$$

Then

$$\boxed{\begin{aligned} y - 2 &= \frac{-8}{9}(x-1) \\ &= \frac{-8}{9}x + \frac{8}{9} \end{aligned}} \quad (1)$$

or

$$\boxed{y = \frac{-8}{9}x + \frac{26}{9}}$$