MATH 135: Quiz 10

/10

November 11, 2014

Name: Solutions Sec: ____________

1. Compute the following limits.

(a)

/2

$$\lim_{x\to 3} \frac{x^3-27}{x^2+x-6}$$

$3^2+3-6 = 9+3-6 = 6 \neq 0$, So this is (1)
Not an indeterminate form. Thus

$$\lim_{x\to 3} \frac{x^3-27}{x^2+x-6} = \frac{3^3-27}{3^2+3-6} = \frac{0}{6} = \boxed{0} \quad (1)$$

(b)

/3

$$\lim_{x\to \pi} \frac{\sin x}{\pi^2-x^2}$$

$\sin \pi = 0$, So this is of the form $\frac{0}{0}$. (1)

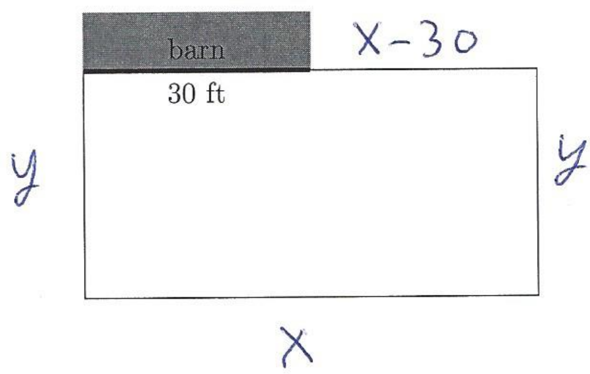
Applying L'Hôpital's Rule (1)

$$\lim_{x\to\pi} \frac{\sin x}{\pi^2-x^2} = \lim_{x\to\pi} \frac{(\sin x)'}{(\pi^2-x^2)'} = \lim_{x\to\pi} \frac{\cos x}{-2x}$$

$$= \frac{\cos \pi}{-2\pi} = \boxed{+\frac{1}{2\pi}} \quad (1)$$

/5

2. A farmer has 170 feet of fencing with which to enclose a pen for his sheep. He wants to build the pen adjacent to a 30 ft long barn, and will not need to put fencing against the barn. What is the area of the largest pen he can build?



Trying to maximize Area $= xy$ (1)

Under the constraint

$$x - 30 + y + x + y = 170$$
$$2x + 2y - 30 = 170$$
$$2x + 2y = 200 \quad (1)$$
$$x + y = 100$$

So $y = 100 - x$. Plugging this into area gives

$$\text{Area} = A(x) = x(100 - x) = 100x - x^2$$

Interval: $x \geq 30$ (so $x - 30 \geq 0$) $\quad \underline{[30, 100]}$ (1)
$y \geq 0 \Rightarrow x \leq 100$

Ignore this

__Optimize__ $A'(x) = 100 - 2x = 0 \Rightarrow x = 50$

(1)

x	$A(x)$
30	$30(100-30) = 30 \cdot 70 = 2100$
50	$50(100-50) = 50 \cdot 50 = 2500$
100	$100(100-100) = 0$

Ignore this

So the maximum area is

$\boxed{2500 \text{ ft}^2}$ (1)