MATH 251, Mariano Echeverria

Problems Optimization

This material corresponds roughly to sections 14.7 and 14.8 in the book.

Problem 1. Find the shortest distance from the plane $3x - 2y - z = 3$ to the origin using Lagrange multipliers.

Problem 2. A rectangular box, open at the top, is to hold 256 cubic centimeters of cat food. Find the dimensions for which the surface area (bottom and four sides) is minimized.

Problem 3. Suppose that the output of a manufacturing firm is a quantity Q of product which is a function K of capital equipment or investment and the amount of labor L used. For example, the Cobb-Douglas production function is $Q(K, L) = AK^{\alpha}L^{1-\alpha}$, where A, α are positive constants and $\alpha < 1$. This is sometimes a simple model for the national economy. If the price of labor is p, the price of capital is q, and the firm can spend no more than B dollars, find the amount of capital and labor which maximizes the output Q. Solve this problem using the Lagrange multiplier method.

Problem 4. Find the constants a, b for which $F(a, b) = \int_0^{\pi} (\sin x - (ax^2 + bx))^2 dx$ is a minimum. Check that these values of a, b do correspond to a minimum.

Problem 5. Find the shortest distance from the origin to the curve of intersection of the surfaces $xyz = a$, $y = bx$ where $a > 0$, $b > 0$.

Problem 6. Prove that the shortest distance from the point (a, b, c) to the plane $Ax + By + Cz + D = 0$ is

$$\left| \frac{Aa + Bb + Cc + D}{\sqrt{A^2 + B^2 + C^2}} \right| \tag{1}$$