MATH 251, Mariano Echeverria

Problems Curves and Geometry in Space

This material corresponds roughly to sections 12.5, 13.1, 13.2 and 13.4 in the book, as well as the study guide "Curves and geometry"

Problem 1. **[this problem will not be evaluated on the exam, only on the written homework ☺]** Let $\mathbf{r}(t) = a\cos t\mathbf{i} + b\sin t\mathbf{j}$ for $0 \leq t \leq 2\pi$ parameterize the ellipse $x^2/a^2 + y^2/b^2 = 1$.

a) Compute the curvature $\kappa(t)$ of the curve.

b) Suppose that $a > b$. When is the curvature maximal? Try to think geometrically why this must be the case.

Problem 2. **[this problem will not be evaluated on the exam, only on the written homework ☺]**

Notice that the graph of a function $y = f(x)$ can be considered as a curve on the xy plane

$$\mathbf{r}(x) = (x, f(x)) \tag{1}$$

Show that the curvature of this curve is always

$$\kappa(x) = \frac{|f''(x)|}{(1 + (f')^2)^{3/2}} \tag{2}$$

Problem 3. **[this problem will not be evaluated on the exam, only on the written homework ☺]**

Compute the curvature κ for the twisted cubic

$$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k} \tag{3}$$

Problem 4. Let $\mathbf{r}(t)$ represent the trajectory of a curve. Show that at a local maximum or minimum of $f(t) = \|\mathbf{r}(t)\|$, we have that $\mathbf{r}(t)$ is perpendicular to $\frac{d}{dt}\mathbf{r}(t)$. Hint: see also Kepler's problem on the written assignment.