M A T H E M A T I C A L P H Y S I C S S E M I N A R

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Hydrodynamic Turbulence as a Problem in Non-equilibrium Statistical Mechanics

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Abstract

The problem of hydrodynamic turbulence is reformulated as a heat flow problem along a chain of mechanical systems which describe units of fluid of smaller and smaller spatial extent. These units are macroscopic but have few degrees of freedom, and can be studied by the methods of (microscopic) non-equilibrium statistical mechanics. The fluctuations predicted by statistical mechanics correspond to the intermittency observed in turbulent flows. Specifically, we obtain the formula

$$\zeta_p = \frac{p}{3} - \frac{1}{\ln \kappa} \ln \Gamma(\frac{p}{3} + 1)$$

for the exponents of the structure functions ($\langle |\Delta_r v|^p \rangle \sim r^{\zeta_p}$). The meaning of the adjustable parameter κ is that when an eddy of size r has decayed to eddies of size r/κ their energies have a thermal distribution. The above formula, with $(\ln \kappa)^{-1} = .32 \pm .01$ is in good agreement with experimental data. This lends support to our physical picture of turbulence, a picture which can thus also be used in related problems.