

Rutgers University
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**Stable Big Bang Formation in Near-FLRW Solutions to the
Einstein-Scalar Field System**

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Abstract

I will discuss some results that I recently obtained in collaboration with Igor Rodnianski. Our main result is a proof of stable Big Bang formation in small perturbations of the well-known (FLRW) solution to the Einstein-scalar field system. The FLRW solution is a special case of a family of spatially homogeneous, isotropic solutions that arise in cosmology. It models a scalar field evolving in a spacetime that expands as $t \rightarrow \infty$ and that collapses as $t \downarrow 0$. In particular, the FLRW solution contains a “Big Bang” singularity at $\Sigma_0 := \{t = 0\}$. To study the perturbed solutions, we place data on a Cauchy hypersurface Σ_1 that are close to the FLRW data (at time 1) as measured by a Sobolev norm. No symmetry assumptions are made on the data. We then study the global behavior of the perturbed solution in the *collapsing* direction. We first show that the spacetime region of interest can be foliated by a family of spacelike Cauchy hypersurfaces Σ_t , $t \in (0, 1]$, of constant mean curvature $-\frac{1}{3}t^{-1}$. We then analyze the behavior of the solution as $t \downarrow 0$ and provide a detailed description of its asymptotics. Our main conclusion is that the perturbed solution remains globally close to the FLRW solution and has approximately monotonic behavior. In particular, the perturbed solution also has a Big Bang singularity at Σ_0 . More precisely, as $t \downarrow 0$, various curvature invariants uniformly blow-up and the volume of Σ_t collapses to 0. These blow-up results demonstrate the validity of Penrose’s Strong Cosmic Censorship conjecture for the past half of the perturbed spacetimes. We have also shown that the same results hold for the stiff fluid matter model.

From the point of view of analysis, our main results can be viewed as a proof of stable blow-up for an open set of solutions to a highly nonlinear elliptic-hyperbolic system. The most important aspect of our analysis is our identification of a new L^2 -type *energy almost-monotonicity inequality* that holds for the solutions under consideration.