Some wonderful conjectures (but almost no theorems) at the boundary between analysis, combinatorics and probability: The entire function $F(x,y) = \sum_{n=0}^{\infty} \frac{x^n}{n!} y^{n(n-1)/2}$, the polynomials $P_N(x,y) = \sum_{n=0}^{N} \binom{N}{n} x^n y^{n(N-n)}$, and the generating polynomials of connected graphs

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Abstract

I discuss some analytic and combinatorial properties of the entire function $F(x,y) = \sum_{n=0}^{\infty} \frac{x^n}{n!} y^{n(n-1)/2}$. This function (or formal power series) arises in numerous problems in enumerative combinatorics, notably in the enumeration of connected graphs, and in statistical mechanics in connection with the Potts model on the complete graph ("mean-field" or Curie–Weiss Potts model). This circle of problems also touches on the theory of integrable systems in classical mechanics (Calogero–Moser system) and on identities for q-series. Some of this talk can be found at http://algo.inria.fr/seminars/sem09-10/sokal-slides.pdf