

M A T H E M A T I C A L P H Y S I C S S E M I N A R

Rutgers University
Hill Center - Room 705

Hausdorff Dimension, Iterated Function Systems and Positive Operators

Roger Nussbaum, Rutgers University
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Abstract

For certain classes of “iterated function systems” there is a naturally associated fractal object. We are interested in the Hausdorff dimension of this fractal object and in how this Hausdorff dimension can be rigorously estimated. In contrast to most of the literature, we shall avoid the use of thermodynamic formalism. When the functions in the iterated function system are “infinitesimal similitudes”, this problem is connected with the theory of certain positive linear operators, with differentiability properties of positive eigenvectors of these operators and with explicit estimates on the derivatives of the eigenvectors. If $B = b = m + ni : m \geq 1, m \text{ and } n \text{ integers, } i = \text{square root of } -1$ and $g_b(z) = 1/(z + b)$ for complex numbers z , there is a fractal object f associated to the iterated function system $\{g_b : b \in B\}$. As an example, we shall sketch (joint work with Rick Falk) how to prove that d , the Hausdorff dimension of F , satisfies $1.854 < d < 1.857$, which greatly improves earlier estimates of Mauldin and Urbanski.