M A T H E M A T I C A L P H Y S I C S S E M I N A R

Rutgers University
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Trends to equilibrium in Boltzmann's equation via Kac's model and the entropy method

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Abstract

One of the most influential equations in the kinetic theory of gases is the so-called Boltzmann equation. While used widely in practice, its irreversibility poses a huge mathematical difficulty as one assumes that such equation arises in Newtonian settings where the laws are all reversible. This raises an interesting mathematical question: Can one achieve an irreversible system from a reversible one, in macroscopic time scales? If so, how can it happen? In his 1956 paper, Marc Kac presented a method to achieve such a thing by considering the limit of a many particle jump process. Kac considered a model of N indistinguishable particles, with one dimensional velocities, that undergo a random binary collision process. Under a special property, called chaoticity, Kac managed to show that when one takes the number of particles to infinity, the limit of the first marginal of the N-particle distribution function satisfies a caricature of the Boltzmann equation. Kac's hope was that using his model and taking the number of particles to infinity, one can learn properties of the Boltzmann equation - specifically, the rate of convergence to equilibrium. In our talk we will mention the spatially homogeneous Boltzmann equation and describe Kac's model. We will discuss the so-called spectral gap problem, posed by Kac in hope to achieve an exponential rate of decay to equilibrium in the Boltzmann equation, and show why that method has failed. We will then discuss the entropy and entropy-entropy production ratio problem, extending Kac's spectral gap problem to a more natural

functional, and explain why in its full generality this method is still inadequate to deal with the issue. We will also mention McKean model, an extension of Kac's model where the velocities of the particles are allowed to be d-dimensional, and mention why even in this more realistic setting the entropy-entropy production ratio doesn't yield a better result. If we'll have some time remaining we will briefly discuss possible connections between the problem at hand and the concept of entropic chaos, one that plays in interesting role on the entropy-entropy production problem.