

THOMAS- FERMI REVISITED

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ABSTRACT. Thomas - Fermi equation extensively studied by E. Lieb and B. Simon (Adv. Math. 1977) involves a nonlinear PDE of the form

$$(1) \quad -\Delta u + u^{3/2} = \sum m_i \delta_{a_i} \text{ in } \mathbb{R}^3$$

where δ_a denotes the Dirac mass at a , and $m_i > 0$. More generally, we will investigate nonlinear PDE's of the form

$$(2) \quad -\Delta u + u^p = \mu \text{ in } \mathbb{R}^n$$

or

$$(3) \quad -\Delta u + (e^u - 1) = \mu, \text{ where } \mu \text{ is a measure in } \mathbb{R}^n.$$

While (1) admits a solution for every measure on the right-hand side, (2) or (3) need not always have solutions for a general measure μ .

In cases where existence fails I will explain how solutions may “disappear” and I will propose a natural concept of generalized solution.