On the relativistic Q.M. of N-body electron-photon systems in 1+1 dimensions

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Introduction

Using a relativistic extension of Quantum Mechanics known as Multi-time Wave Function formulation, we examine an N-body quantum mechanical system of photons and electrons that interact only upon contact, in one space dimension. For the case N=1 we explore the trajectories of non-interacting free photons and electrons. For the case N=2 we attempt to gain a qualitative understanding of Compton Scattering for this system. For N=3 we investigate the case in which two electrons are separated by a photon, with interactions given by a no-crossing boundary condition. We describe the way in which data propagates in this system and what happens to this data when it hits a boundary.

Single-Body Trajectories

We treat the photon as a relativistic quantum particle and require that its wave-function satisfies the following transport equation:

$$-i\hbar D\Psi_{\mathrm{ph}} = 0, \qquad D := \gamma^\mu \partial_\mu. \tag{1}$$

The trajectories can be derived from the wave-function by solving a first-order guiding equation.

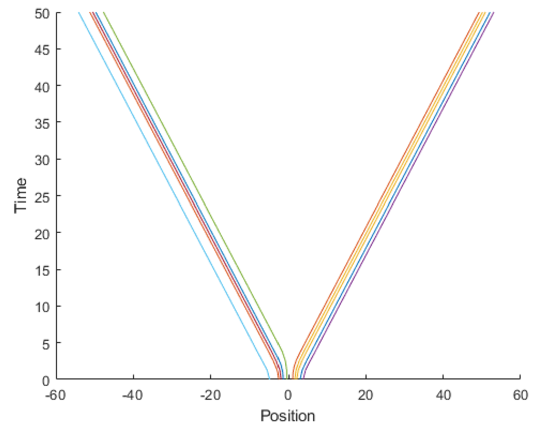

Fig. 1: Various Photon Trajectories

We treat the electron similarly, except that it must instead satisfy the following massive Dirac equation:

$$-i\hbar D\Psi_{\mathrm{e}} + m_{\mathrm{e}}\Psi_{\mathrm{e}} = 0 \tag{2}$$

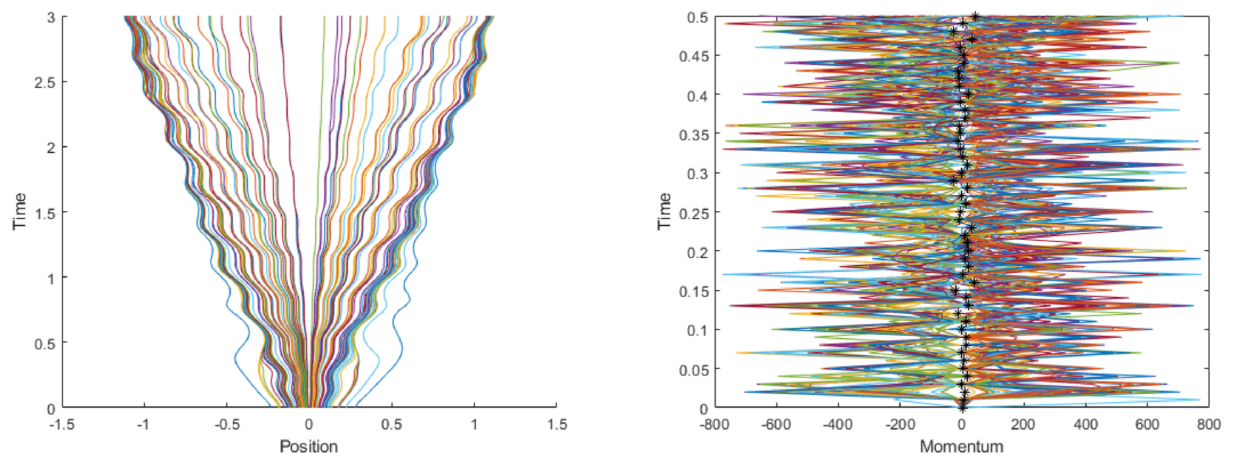

Fig. 2: Various Electron Trajectories (left) and the Momentum vs Time for Various Initial Positions (right)

Two-Body Interacting System

To obtain an interacting system from a non-interacting system, it is necessary to add a condition that prevents the particles from simply going through each other. This is done by adding a boundary condition on the coincidence set [1]: the relative velocity of photon and electron is set to be 0 whenever the two particles are at the same space and time point. These are the trajectories of the interacting electron-photon system with the boundary condition added in. The solid lines show the interacting system trajectories and the dotted lines show the non-interacting trajectories.

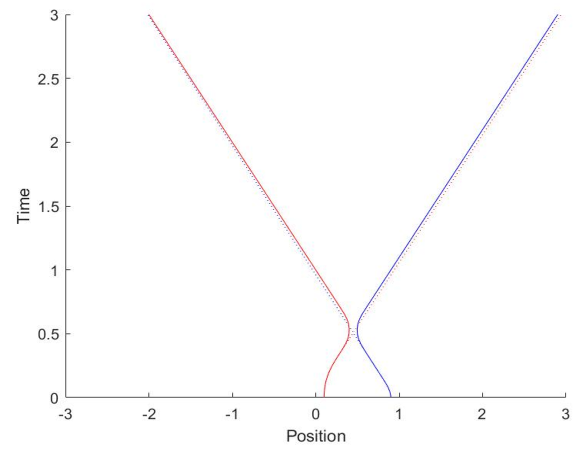

Fig. 3: Interacting Photon (Red) and Electron (Blue) Trajectories

Assuming that positive plane energy waves go in and out of the scattering zone, we find that the frequency of the final photon is lower than the frequency of the initial photon, and that the difference in the photon momentum and energy is transferred to the electron. In other words,

$$k_{el}^{out} - k_{el}^{in} = k_{ph}^{in} - k_{ph}^{out} = 2k_{ph}^{in}\left(\frac{\omega + k_{ph}^{in}}{\omega + 2k_{ph}^{in}}\right) \tag{3}$$

The following graph shows that the electron momentum varies linearly with respect to initial photon frequency.

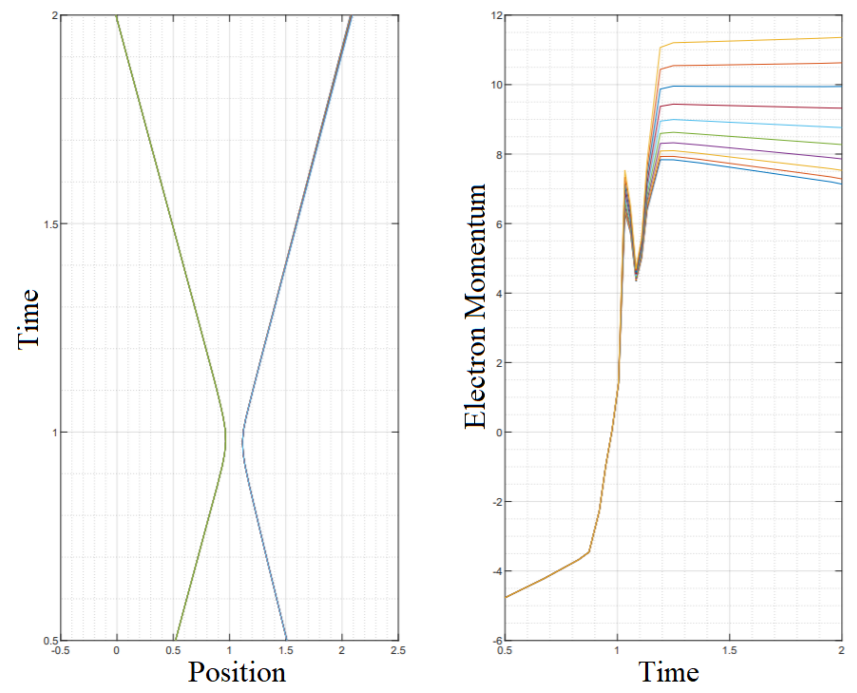

Fig. 4: Change in electron momentum due to scattering. The various colors on the right graph correspond to various initial frequencies for the photon.

Three-Body System

For a system with a photon between two electrons, we require that our wave-function satisfies a transport in the photon's variables, and two Dirac equations in the first and second electron's variables.

$$\begin{cases} -i\hbar D_{\mathrm{ph}}\Psi = 0 \\ -i\hbar D_{\mathrm{e}_1}\Psi + m_e\Psi = 0 \\ -i\hbar D_{\mathrm{e}_2}\Psi + m_e\Psi = 0 \text{ in } \mathcal{S}_1 \end{cases} \tag{4}$$

The following figures show the propagation of our data in single-time configuration space.

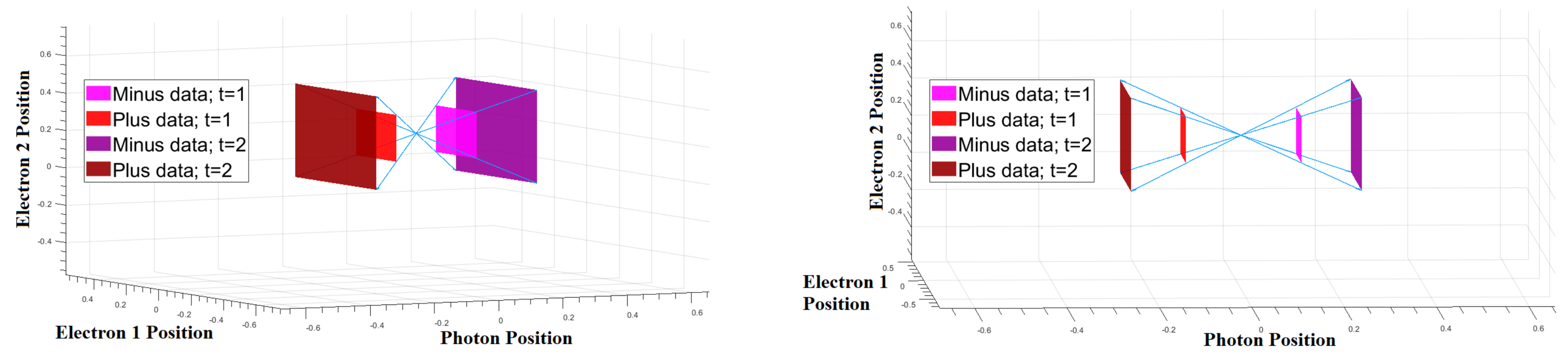

Fig. 5: Forward Propagation of Minus and Plus Data in the Noninteracting Case

To make our system interacting, we add in the following boundary conditions, which are equivalent to forbidding the photon from passing through either electron.

$$\begin{cases} \psi_{-+\varsigma_3} = e^{i\theta}\sqrt{\dfrac{X^0+X^1}{X^0-X^1}}\,\psi_{+-\varsigma_3} & \text{on } \mathcal{C}_1 \\ \psi_{-\varsigma_2+} = e^{-i\theta}\sqrt{\dfrac{X^0+X^1}{X^0-X^1}}\,\psi_{+\varsigma_2-} & \text{on } \mathcal{C}_2 \end{cases} \tag{5}$$

The following figures display the effect this condition has when data hits the boundary–new data, which propagates in the opposite direction, is born.

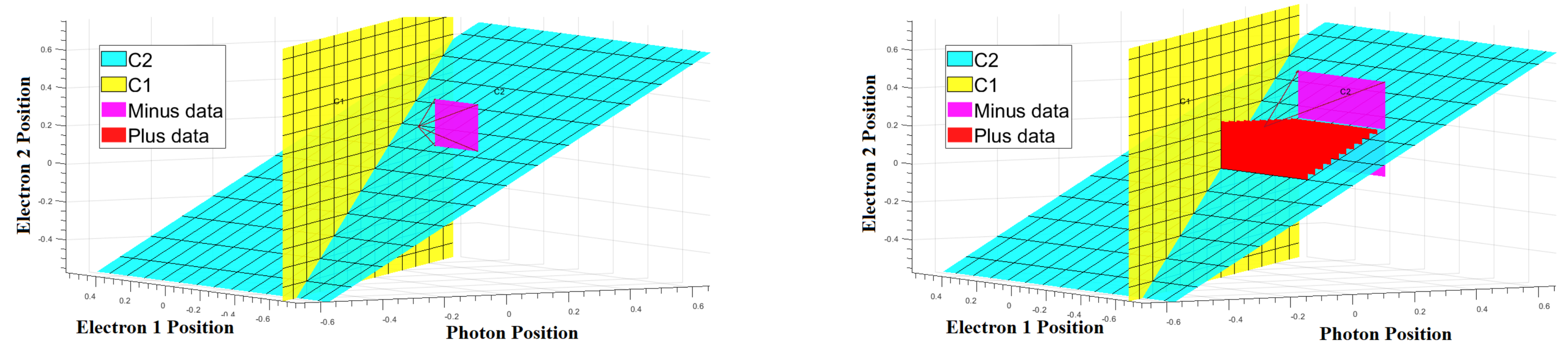

Fig. 6: Forward Propagation of Minus Data in the Interacting Case

References

[1] Kiessling, M., Lienert, M., & Tahvildar-Zadeh, A. S., "A Lorentz-covariant interacting electron-photon system in one space dimension," *Lett. Math. Phys.* **110** (3153–3195) (2020)

[2] Kiessling, M. K.-H., and Tahvildar-Zadeh, A. S., "On the quantum mechanics of a single photon," *J. Math. Phys.*, **59**:112302 (2018).