Point singularities in classical field theory

Lawrence Frolov

Department of Mathematics
Rutgers, The State University of New Jersey

August 5th, 2025

What is a Classical Point Singularity?

- On $n+1$ dimensional space-time $\mathbb{R}^{1,n} \cong \mathbb{R} \times \mathbb{R}^n$ with metric $\eta = \text{diag}(1, -1, -1 \dots)$ and coordinates $x = (x^0, x^1, x^2, \dots)$.

What is a Classical Point Singularity?

- On $n+1$ dimensional space-time $\mathbb{R}^{1,n} \cong \mathbb{R} \times \mathbb{R}^n$ with metric $\eta = \text{diag}(1, -1, -1...)$ and coordinates $x = (x^0, x^1, x^2, ...)$.
- A point charge is represented by a future directed time-like curve ("world-line") $z(\theta) : \mathbb{R} \to \mathbb{R}^{1,n}$, $\frac{dz^0}{d\theta} > |\frac{d\vec{z}}{d\theta}|$.

What is a Classical Point Singularity?

- On $n+1$ dimensional space-time $\mathbb{R}^{1,n} \cong \mathbb{R} \times \mathbb{R}^n$ with metric $\eta = \mathrm{diag}(1, -1, -1...)$ and coordinates $x = (x^0, x^1, x^2, ...)$.
- A point charge is represented by a future directed time-like curve ("world-line") $z(\theta) : \mathbb{R} \to \mathbb{R}^{1,n}$, $\frac{dz^0}{d\theta} > |\frac{d\vec{z}}{d\theta}|$.
- We will use the proper time parameterization $z(\tau)$ defined by $d\tau = \sqrt{\eta_{\mu\nu}\dot{z}^\mu \dot{z}^\nu} d\theta$.

What is a Classical Point Singularity?

- On $n+1$ dimensional space-time $\mathbb{R}^{1,n} \cong \mathbb{R} \times \mathbb{R}^n$ with metric $\eta = \text{diag}(1, -1, -1...)$ and coordinates $x = (x^0, x^1, x^2, ...)$.
- A point charge is represented by a future directed time-like curve ("world-line") $z(\theta) : \mathbb{R} \to \mathbb{R}^{1,n}$, $\frac{dz^0}{d\theta} > |\frac{d\vec{z}}{d\theta}|$.
- We will use the proper time parameterization $z(\tau)$ defined by $d\tau = \sqrt{\eta_{\mu\nu}\dot{z}^\mu \dot{z}^\nu} d\theta$.
- Associated to the particle is a mass $m > 0$, scalar charge $\alpha \in \mathbb{R}$, velocity $u^\mu := \frac{dz^\mu}{d\tau}$, and momentum $p^\mu \propto u^\mu$.

What is a Classical Point Singularity?

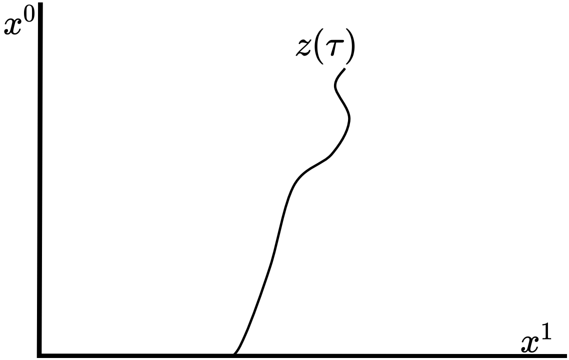
- On $n+1$ dimensional space-time $\mathbb{R}^{1,n} \cong \mathbb{R} \times \mathbb{R}^n$ with metric $\eta = \text{diag}(1, -1, -1 \ldots)$ and coordinates $x = (x^0, x^1, x^2, \ldots)$.
- A point charge is represented by a future directed time-like curve ("world-line") $z(\theta) : \mathbb{R} \to \mathbb{R}^{1,n}$, $\frac{dz^0}{d\theta} > |\frac{d\vec{z}}{d\theta}|$.
- We will use the proper time parameterization $z(\tau)$ defined by $d\tau = \sqrt{\eta_{\mu\nu}\dot{z}^\mu \dot{z}^\nu} d\theta$.
- Associated to the particle is a mass $m > 0$, scalar charge $\alpha \in \mathbb{R}$, velocity $u^\mu := \frac{dz^\mu}{d\tau}$, and momentum $p^\mu \propto u^\mu$.
- Notation: Jump and Average:

$$\overline{f}(z) = \lim_{\epsilon \to 0} \frac{f(z+\epsilon) + f(z-\epsilon)}{2}, \quad [f]_{x=z} = \lim_{\epsilon \to 0^+} \left(f(z+\epsilon) - f(z-\epsilon)\right) \tag{1}$$

Scalar Point Charge

The scalar point charge acts as a source for the scalar field via

$$\Box U := \partial_0^2 U - \Delta U = \alpha \int \delta^{(n+1)}(x - z(\tau)) d\tau \tag{2}$$



Motion of Scalar Field Animation

Given initial data (V_0, V_1) and a charge trajectory $z(\tau)$, the solution for U is well-known.

$$\begin{cases} \Box U = \alpha \int \delta^{(n+1)}(x - z(\tau))d\tau \\ U\big|_{x^0=0} = V_0 \\ \partial_0 U\big|_{x^0=0} = V_1 \end{cases} \tag{3}$$

Figure: Animation of scalar field sourced by a moving point charge in one space dimension.

(In)admissible Force Laws

- We demand a joint evolution which preserves the total energy-momentum of the field-particle system.

(In)admissible Force Laws

- We demand a joint evolution which preserves the total energy-momentum of the field-particle system.

$$\partial_\mu (T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{field}}) = 0 \tag{4}$$

(In)admissible Force Laws

- We demand a joint evolution which preserves the total energy-momentum of the field-particle system.

$$\partial_\mu(T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{field}}) = 0 \tag{4}$$

- For *fluids* of sufficient regularity, cons of En-Mom returns an equation of motion for the fluid.

(In)admissible Force Laws

- We demand a joint evolution which preserves the total energy-momentum of the field-particle system.

$$\partial_\mu (T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{field}}) = 0 \tag{4}$$

- For *fluids* of sufficient regularity, cons of En-Mom returns an equation of motion for the fluid.
- It is often claimed that point singularities satisfy the EQ of motion for fluid test-particles:

$$\frac{dp^\mu}{d\tau} = -\alpha \partial^\mu U(z) \tag{5}$$

(In)admissible Force Laws

- We demand a joint evolution which preserves the total energy-momentum of the field-particle system.

$$\partial_\mu (T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{field}}) = 0 \tag{4}$$

- For *fluids* of sufficient regularity, cons of En-Mom returns an equation of motion for the fluid.
- It is often claimed that point singularities satisfy the EQ of motion for fluid test-particles:

$$\frac{dp^\mu}{d\tau} = -\alpha \partial^\mu U(z) \tag{5}$$

- $\partial^\mu U$ cannot be evaluated along $z(\tau)$, joint evolution equations are inconsistent.

Inconsistency

- It is sometimes claimed that we should ignore the singular contribution of a particle to its own force law. This ignores *radiation-reaction*.

Inconsistency

- It is sometimes claimed that we should ignore the singular contribution of a particle to its own force law. This ignores *radiation-reaction*.
- Dirac [5] famously tried to derive an EQ of motion directly from $\partial_\mu T^{\mu\nu}_{\text{matter}} = -\partial_\mu T^{\mu\nu}_{\text{field}}$.

Inconsistency

- It is sometimes claimed that we should ignore the singular contribution of a particle to its own force law. This ignores *radiation-reaction*.
- Dirac [5] famously tried to derive an EQ of motion directly from $\partial_\mu T^{\mu\nu}_{\text{matter}} = -\partial_\mu T^{\mu\nu}_{\text{field}}$.
- Problem: En-Mom of field sourced by point singularity in $\mathbb{R}^3$ is changing at divergent rate for all time, $T^{\mu\nu}_{\text{field}} \approx \frac{1}{r^4}$

Inconsistency

- It is sometimes claimed that we should ignore the singular contribution of a particle to its own force law. This ignores *radiation-reaction*.
- Dirac [5] famously tried to derive an EQ of motion directly from $\partial_\mu T^{\mu\nu}_{\text{matter}} = -\partial_\mu T^{\mu\nu}_{\text{field}}$.
- Problem: En-Mom of field sourced by point singularity in $\mathbb{R}^3$ is changing at divergent rate for all time, $T^{\mu\nu}_{\text{field}} \approx \frac{1}{r^4}$
- Dirac invented *infinite bare-mass renormalization* to recover an equation of motion for the point singularity.

Inconsistency

- It is sometimes claimed that we should ignore the singular contribution of a particle to its own force law. This ignores *radiation-reaction*.
- Dirac [5] famously tried to derive an EQ of motion directly from $\partial_\mu T^{\mu\nu}_{\text{matter}} = -\partial_\mu T^{\mu\nu}_{\text{field}}$.
- Problem: En-Mom of field sourced by point singularity in $\mathbb{R}^3$ is changing at divergent rate for all time, $T^{\mu\nu}_{\text{field}} \approx \frac{1}{r^4}$
- Dirac invented *infinite bare-mass renormalization* to recover an equation of motion for the point singularity.
- Problem: Using Colombeau Algebra, Gsponer(2008) showed that one can derive a large family of different particle force laws.

Quantum Mechanical Cop-out

- Shouldn't we be modeling point particles using quantum theory?

Quantum Mechanical Cop-out

- Shouldn't we be modeling point particles using quantum theory?
- QFT famously admits divergent quantities that can be traced back to divergences in the classical theory of relativistic point charges.

Quantum Mechanical Cop-out

- Shouldn't we be modeling point particles using quantum theory?
- QFT famously admits divergent quantities that can be traced back to divergences in the classical theory of relativistic point charges.
- Ex: Annihilation of quantum particle at origin in $\mathbb{R}^3$.

Quantum Mechanical Cop-out

- Shouldn't we be modeling point particles using quantum theory?
- QFT famously admits divergent quantities that can be traced back to divergences in the classical theory of relativistic point charges.
- Ex: Annihilation of quantum particle at origin in $\mathbb{R}^3$.

$$\text{Non-Rel:} \quad i\partial_t \psi = \left(-\Delta + \frac{ik}{4\pi} \delta^3(\vec{x}) \right) \psi. \tag{6}$$

Quantum Mechanical Cop-out

- Shouldn't we be modeling point particles using quantum theory?
- QFT famously admits divergent quantities that can be traced back to divergences in the classical theory of relativistic point charges.
- Ex: Annihilation of quantum particle at origin in $\mathbb{R}^3$.

$$\text{Non-Rel:} \quad i\partial_t \psi = -\Delta\psi, \quad \lim_{r\to 0} \partial_r(r\psi(\vec{x})) = ik \lim_{r\to 0} r\psi(\vec{x}). \tag{6}$$

Quantum Mechanical Cop-out

- Shouldn't we be modeling point particles using quantum theory?
- QFT famously admits divergent quantities that can be traced back to divergences in the classical theory of relativistic point charges.
- Ex: Annihilation of quantum particle at origin in $\mathbb{R}^3$.

$$\text{Non-Rel:} \quad i\partial_t \psi = -\Delta\psi, \quad \lim_{r\to 0} \partial_r(r\psi(\vec{x})) = ik \lim_{r\to 0} r\psi(\vec{x}). \tag{6}$$

$$\text{Rel:} \quad i\partial_t \psi = \vec{\alpha}\cdot\nabla\psi + m\beta\psi, \quad \text{B.C only valid in 1 dim} \tag{7}$$

State of Affairs

- Kiessling[1] proposed in 2019 that the force law should be derived directly from energy-momentum conservation

State of Affairs

- Kiessling[1] proposed in 2019 that the force law should be derived directly from energy-momentum conservation

$$\partial_\mu \left(T_p^{\mu\nu} + T_U^{\mu\nu} \right) = 0 \tag{8}$$

State of Affairs

- Kiessling[1] proposed in 2019 that the force law should be derived directly from energy-momentum conservation

$$\partial_\mu \left(T_p^{\mu\nu} + T_U^{\mu\nu} \right) = 0 \tag{8}$$

$$T_U^{\mu\nu} := (\partial^\mu U)(\partial^\nu U) - \frac{1}{2}\eta^{\mu\nu}(\partial_\mu U)(\partial^\mu U). \tag{9}$$

State of Affairs

- Kiessling[1] proposed in 2019 that the force law should be derived directly from energy-momentum conservation

$$\partial_\mu \left(T_p^{\mu\nu} + T_U^{\mu\nu} \right) = 0 \tag{8}$$

$$T_U^{\mu\nu} := (\partial^\mu U)(\partial^\nu U) - \frac{1}{2}\eta^{\mu\nu}(\partial_\mu U)(\partial^\mu U). \tag{9}$$

$$T_p^{\mu\nu} := \int u^\mu p^\nu \delta^{(n+1)}(x - z(\tau))d\tau \tag{10}$$

- In dimensions $n > 1$, we expect $\Box U = \alpha \int \delta^{(n+1)}(x - z(\tau))d\tau$ to be inconsistent with energy-momentum conservation [1,4].

State of Affairs

- Kiessling[1] proposed in 2019 that the force law should be derived directly from energy-momentum conservation

$$\partial_\mu \left(T_p^{\mu\nu} + T_U^{\mu\nu} \right) = 0 \tag{8}$$

$$T_U^{\mu\nu} := (\partial^\mu U)(\partial^\nu U) - \frac{1}{2}\eta^{\mu\nu}(\partial_\mu U)(\partial^\mu U). \tag{9}$$

$$T_p^{\mu\nu} := \int u^\mu p^\nu \delta^{(n+1)}(x - z(\tau))d\tau \tag{10}$$

- In dimensions $n > 1$, we expect $\Box U = \alpha \int \delta^{(n+1)}(x - z(\tau))d\tau$ to be inconsistent with energy-momentum conservation [1,4].
- Modify "vacuum law" to regularize sourced fields:

$$(1 + \frac{1}{\kappa^2}\Box)\Box U = \alpha \int \delta^{(4)}(x - z(\tau))d\tau \qquad \text{K-T.Z[2], Hoang et al.[3]} \tag{11}$$

Conservation of Stress Energy-Momentum

Theorem (Force Law in 1 Space Dimension)

Suppose U satisfies $\square U = \alpha \int \delta^{(2)}(x - z(\tau))d\tau$ on $\mathbb{R}^{1,1}$ where $z(\tau)$ is an arbitrary time-like world-line.

Conservation of Stress Energy-Momentum

Theorem (Force Law in 1 Space Dimension)

Suppose U satisfies $\square U = \alpha \int \delta^{(2)}(x - z(\tau)) d\tau$ *on* $\mathbb{R}^{1,1}$ *where* $z(\tau)$ *is an arbitrary time-like world-line. Then assuming conservation of energy-momentum*

$$\partial_\mu T_p^{\mu\nu} = -\partial_\mu T_U^{\mu\nu}, \quad \nu \in \{0, 1\} \tag{12}$$

returns the unique force law

Conservation of Stress Energy-Momentum

Theorem (Force Law in 1 Space Dimension)

Suppose U satisfies $\square U = \alpha \int \delta^{(2)}(x - z(\tau))d\tau$ *on* $\mathbb{R}^{1,1}$ *where* $z(\tau)$ *is an arbitrary time-like world-line. Then assuming conservation of energy-momentum*

$$\partial_\mu T_p^{\mu\nu} = -\partial_\mu T_U^{\mu\nu}, \quad \nu \in \{0, 1\} \tag{12}$$

returns the unique force law

$$\frac{dp^\nu}{d\tau} = -\left[n_\mu T_U^{\mu\nu}(z^0, x^1)\right]_{x^1 = z^1}. \tag{13}$$

Conservation of Stress Energy-Momentum

Theorem (Force Law in 1 Space Dimension)

Suppose U satisfies $\Box U = \alpha \int \delta^{(2)}(x - z(\tau))d\tau$ on $\mathbb{R}^{1,1}$ where $z(\tau)$ is an arbitrary time-like world-line. Then assuming conservation of energy-momentum

$$\partial_\mu T_p^{\mu\nu} = -\partial_\mu T_U^{\mu\nu}, \quad \nu \in \{0, 1\} \tag{12}$$

returns the unique force law

$$\frac{dp^\nu}{d\tau} = -\left[n_\mu T_U^{\mu\nu}(z^0, x^1)\right]_{x^1 = z^1}. \tag{13}$$

This force can be explicitly computed as

$$\frac{dp^\nu}{d\tau} = -\alpha \overline{\partial^\nu U}. \tag{14}$$

Derivation of Force Law 1

$$\int_{\Omega} \partial_{\mu} T^{\mu\nu} dV = \int_{\partial\Omega} T^{\mu\nu} N_{\mu} dS \tag{15}$$

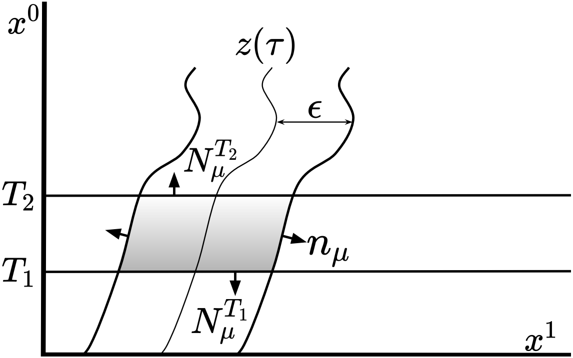
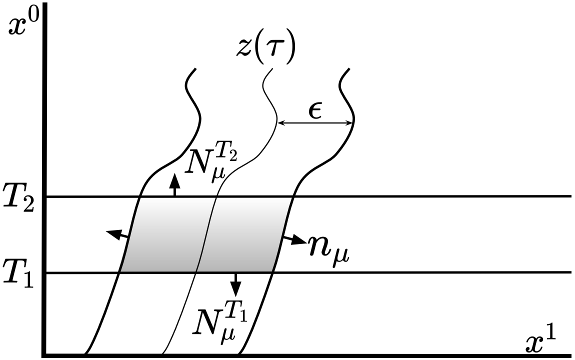


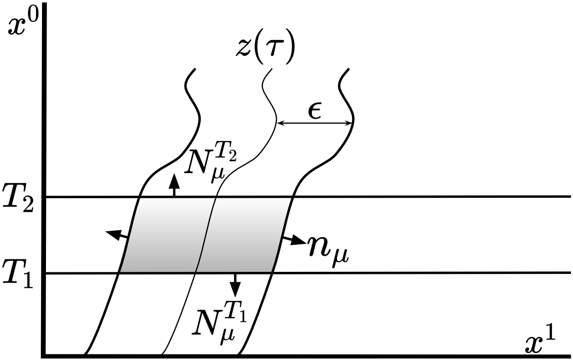
Figure: Region of integration Ω and its normals

$$\int_{\partial\Omega} T_p^{\mu\nu} N_\mu dS = p^\nu(\tau_2) - p^\nu(\tau_1) = \int_{\tau_1}^{\tau_2} \frac{dp^\nu}{d\tau} d\tau. \tag{15}$$



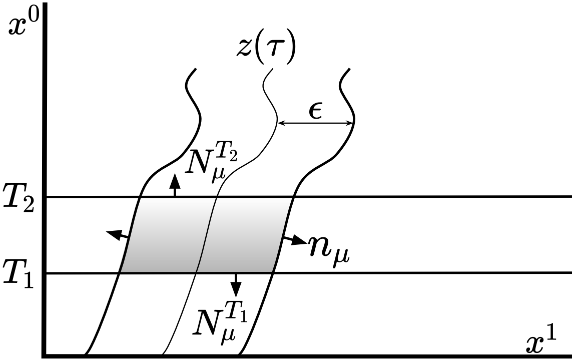
Derivation of Force Law 1

$$\int_{\tau_1}^{\tau_2} \frac{dp^\nu}{d\tau} d\tau = - \int_{\partial\Omega} T_U^{\mu\nu} N_\mu dS. \tag{15}$$



Derivation of Force Law 1

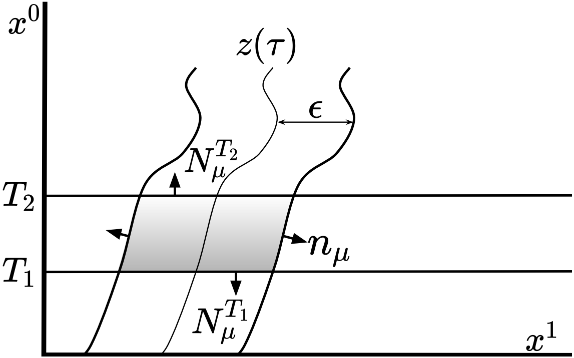
$$\int_{\tau_1}^{\tau_2} \frac{dp^\nu}{d\tau} d\tau = -\lim_{\epsilon \to 0} \int_{\partial\Omega} T_U^{\mu\nu} N_\mu dS. \quad (15)$$



Derivation of Force Law 2

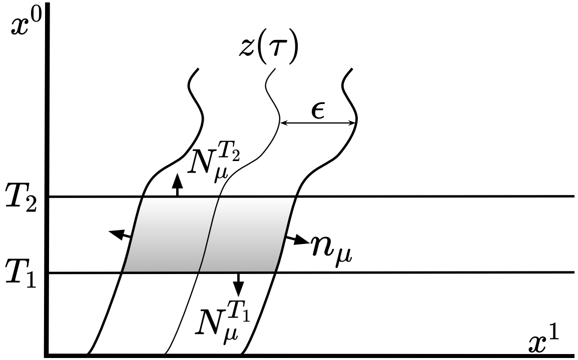
Lemma

Suppose U satisfies equation (3) with $(U, \partial_0 U)\big|_{x^0=0} \in C^{0,1}(\mathbb{R}) \times L^\infty(\mathbb{R})$ and singular source concentrated on a time-like world-line $z(\tau)$. Then U is of regularity $C^{0,1}(\mathbb{R}^{1,1})$, and $\partial_\mu U$, $T_U^{\mu\nu} \in L^\infty(\mathbb{R}^{1,1})$.



Derivation of Force Law 2

$$\lim_{\epsilon \to 0} \int_{\partial\Omega} T_U^{\mu\nu} N_\mu dS = \int_{\tau_1}^{\tau_2} \left[T_U^{\mu\nu} n_\mu(z^0, x^1) \right]_{x^1 = z^1(\tau)} d\tau \tag{16}$$



Analyzing Force Law

So weak conservation of energy-momentum returns

Analyzing Force Law

So weak conservation of energy-momentum returns

$$\int_{\tau_1}^{\tau_2} \frac{dp^\nu}{d\tau} d\tau = - \int_{\tau_1}^{\tau_2} \left[n_\mu T_U^{\mu\nu}(z^0, x^1) \right]_{x^1 = z^1(\tau)} d\tau. \tag{17}$$

as desired.

Analyzing Force Law

So weak conservation of energy-momentum returns

$$\int_{\tau_1}^{\tau_2} \frac{dp^\nu}{d\tau} d\tau = -\int_{\tau_1}^{\tau_2} \left[n_\mu T_U^{\mu\nu}(z^0, x^1) \right]_{x^1 = z^1(\tau)} d\tau. \tag{17}$$

as desired.

- For $(U, \partial_0 U)\big|_{x^0=0}$ smooth away from $z^1(0)$, we have

$$[\partial_\mu U]_{x^1 = z^1(\tau)} = -\alpha n_\mu \tag{18}$$

Analyzing Force Law

So weak conservation of energy-momentum returns

$$\int_{\tau_1}^{\tau_2} \frac{dp^\nu}{d\tau} d\tau = -\int_{\tau_1}^{\tau_2} \left[n_\mu T_U^{\mu\nu}(z^0, x^1) \right]_{x^1 = z^1(\tau)} d\tau. \tag{17}$$

as desired.

- For $(U, \partial_0 U)\big|_{x^0=0}$ smooth away from $z^1(0)$, we have

$$[\partial_\mu U]_{x^1 = z^1(\tau)} = -\alpha n_\mu \tag{18}$$

- The final force law $\frac{dp^\nu}{d\tau} = -\alpha \overline{\partial^\nu U}$ follows from

$$\left[\frac{f^2(x^1)}{2} \right]_{x^1 = z^1} = [f]_{x^1 = z^1} \overline{f} \tag{19}$$

Force Law Consequences 1

Corollary

The force contribution of external scalar fields follows the standard law satisfied by test charges.

$$F^{\nu}_{ext} = -\alpha\partial^{\nu} U_{ext}. \tag{20}$$

However, the singular "self-force" can now be determined, and the expression for it guarantees the conservation of the system's total energy-momentum.

$$F^{\nu}_{self} = -\alpha\overline{\partial^{\nu} U}_{source} = \frac{-\alpha^2 u^{\nu}}{2} \tag{21}$$

Force Law Consequences 2

Theorem

For any set of particle parameters with positive bare mass and non-zero real scalar charge, and for any "admissible" initial data, the joint initial value problem given by

$$\left\{ \begin{array}{rcl} \eta^{\mu\nu}\partial_\mu\partial_\nu U(x) & = & a\int \delta^{(2)}(x - z(\tau))d\tau \\ U(0, x^1) & = & V_0(x^1) \\ \partial_0 U(0, x^1) & = & V_1(x^1), \end{array} \right. \tag{22}$$

$$\begin{cases} \frac{dz^\nu}{d\tau} = u^\nu \\ \frac{dp^\nu}{d\tau} = -\left[n_\mu T_U^{\mu\nu}(z^0, x^1)\right]_{x^1 = z^1(\tau)} . \end{cases} \tag{23}$$

admits a unique, global-in-time solution.

Force Law Consequences 3

Theorem

For any set of particle parameters with positive bare mass and non-zero real scalar charge, and for any "admissible" initial data, the joint initial value problem given by

$$\begin{cases} \eta^{\mu\nu}\partial_\mu\partial_\nu U(x) & = & 0 \\ [\partial_\mu U]_{x^1=z^1(\tau)} & = & -\alpha n_\mu \\ U(0,x^1) & = & V_0(x^1) \\ \partial_0 U(0,x^1) & = & V_1(x^1), \end{cases} \tag{24}$$

$$\begin{cases} \frac{dz^\nu}{d\tau} = u^\nu \\ \frac{dp^\nu}{d\tau} = -\left[n_\mu T_U^{\mu\nu}(z^0,x^1)\right]_{x^1=z^1(\tau)}. \end{cases} \tag{25}$$

admits a unique, global-in-time solution.

Summary

- We studied the joint evolution equations for a scalar field and its point charge source in 1 space dimension

$$\Box U = \alpha \int \delta^{(2)}(x - z(\tau) d\tau. \tag{26}$$

Summary

- We studied the joint evolution equations for a scalar field and its point charge source in 1 space dimension
- We rigorously derived an equation of motion for the point charge from weak energy-momentum conservation

$$\frac{dp^\nu}{d\tau} = -\left[n_\mu T^{\mu\nu}(z^0, x^1)\right]_{x^1=z^1}. \tag{26}$$

Summary

- We studied the joint evolution equations for a scalar field and its point charge source in 1 space dimension
- We rigorously derived an equation of motion for the point charge from weak energy-momentum conservation
- The computed force law closely resembles the one satisfied by test charges.

$$F^{\nu}_{\text{self}} = -a\overline{\partial^{\nu} U}_{\text{source}} = \frac{-a^2 u^{\nu}}{2}. \qquad (26)$$

Summary

- We studied the joint evolution equations for a scalar field and its point charge source in 1 space dimension
- We rigorously derived an equation of motion for the point charge from weak energy-momentum conservation
- The computed force law closely resembles the one satisfied by test charges.
- This force law allowed us to prove well-posedness of the joint evolution problem for the field-particle singularity system.

Summary

- We studied the joint evolution equations for a scalar field and its point charge source in 1 space dimension
- We rigorously derived an equation of motion for the point charge from weak energy-momentum conservation
- The computed force law closely resembles the one satisfied by test charges.
- This force law allowed us to prove well-posedness of the joint evolution problem for the field-particle singularity system.
- Singularity in scalar field equation reinterpreted as Boundary Condition along $\{x = z(\tau)\}$.

Outlook

- These results indicate that point charges interacting with fields of sufficient regularity admit well-posed joint evolutions.

Outlook

- These results indicate that point charges interacting with fields of sufficient regularity admit well-posed joint evolutions.
- In higher dimensions we need to study non-linear wave equations coupled to point sources.

Outlook

- These results indicate that point charges interacting with fields of sufficient regularity admit well-posed joint evolutions.
- In higher dimensions we need to study non-linear wave equations coupled to point sources.
- Non-linear modifications may regularize the sourced fields to permit a well-posed joint evolution

$$\partial_\mu \left(\frac{\partial^\mu U}{\sqrt{1 + \partial_\alpha U \partial^\alpha U}} \right) = a \int \delta^{(n+1)}(x - z(\tau)) d\tau. \qquad (26)$$

Outlook

- These results indicate that point charges interacting with fields of sufficient regularity admit well-posed joint evolutions.
- In higher dimensions we need to study non-linear wave equations coupled to point sources.
- Non-linear modifications may regularize the sourced fields to permit a well-posed joint evolution

$$\partial_\mu \left(\frac{\partial^\mu U}{\sqrt{1 + \partial_\alpha U \partial^\alpha U}} \right) = a \int \delta^{(n+1)}(x - z(\tau)) d\tau. \tag{26}$$

- As a first step: Quasi-Linear Schrödinger with Point Singularity for all $d \in \mathbb{N}$

$$i\partial_t \psi = \vec{\nabla} \cdot \left(\frac{\vec{\nabla}\psi}{\sqrt{1 - |\vec{\nabla}\psi|^2}} \right) + ik\delta^{(n)}(\vec{x})\psi. \tag{27}$$

References

- Michael K.-H. Kiessling. *Force on a point charge source of the classical electromagnetic field*. In: Phys. Rev. D 100 (6 2019), p. 065012. doi: 10.1103/PhysRevD.100.065012. url: https://link.aps.org/doi/10.1103/PhysRevD.100.065012.

References

- Michael K.-H. Kiessling. *Force on a point charge source of the classical electromagnetic field*. In: Phys. Rev. D 100 (6 2019), p. 065012. doi: 10.1103/PhysRevD.100.065012. url: https://link.aps.org/doi/10.1103/PhysRevD.100.065012.
- Michael K.-H. Kiessling and A. Shadi Tahvildar-Zadeh. *Bopp-Landé Thomas-Podolsky electrodynamics as an initial value problem*. in preperation, 2024.

References

- Michael K.-H. Kiessling. *Force on a point charge source of the classical electromagnetic field*. In: Phys. Rev. D 100 (6 2019), p. 065012. doi: 10.1103/PhysRevD.100.065012. url: https://link.aps.org/doi/10.1103/PhysRevD.100.065012.
- Michael K.-H. Kiessling and A. Shadi Tahvildar-Zadeh. *Bopp-Landé Thomas-Podolsky electrodynamics as an initial value problem*. in preperation, 2024.
- Vu Hoang et al. *Radiation reaction in higher-order electrodynamics*. In: Journal of Mathematical Physics 62.7 (2021), p. 072901. doi: 10.1063/5.0042690. eprint: https://doi.org/10.1063/5. 0042690. url: https://doi.org/10.1063/5.0042690.

References

- Michael K.-H. Kiessling. *Force on a point charge source of the classical electromagnetic field*. In: Phys. Rev. D 100 (6 2019), p. 065012. doi: 10.1103/PhysRevD.100.065012. url: https://link.aps.org/doi/10.1103/PhysRevD.100.065012.
- Michael K.-H. Kiessling and A. Shadi Tahvildar-Zadeh. *Bopp-Landé Thomas-Podolsky electrodynamics as an initial value problem*. in preperation, 2024.
- Vu Hoang et al. *Radiation reaction in higher-order electrodynamics*. In: Journal of Mathematical Physics 62.7 (2021), p. 072901. doi: 10.1063/5.0042690. eprint: https://doi.org/10.1063/5. 0042690. url: https://doi.org/10.1063/5.0042690.
- Andre Gsponer. *The self-interaction force on an arbitrarily moving point-charge and its energy-momentum radiation rate: A mathematically rigorous derivation of the Lorentz-Dirac equation of motion*. (2008) 35 pp. e-print arXiv:0812.3493

References

- Lawrence Frolov, Samuel Leigh, Shadi Tahvildar-Zadeh; *Joint evolution of a Lorentz-covariant massless scalar field and its point-charge source in one space dimension*. J. Math. Phys. 1 June 2024; 65 (6): 062901. https://doi.org/10.1063/5.0166527

References

- Lawrence Frolov, Samuel Leigh, Shadi Tahvildar-Zadeh; *Joint evolution of a Lorentz-covariant massless scalar field and its point-charge source in one space dimension*. J. Math. Phys. 1 June 2024; 65 (6): 062901. https://doi.org/10.1063/5.0166527
- Paul Adrien Maurice Dirac. *Classical theory of radiating electrons*. Royal Society, 160:148–169,1938.