Math 549: Suggested Exercises for Lectures 22 and 23

References: Bump Chapter 16, see also Spivak Chapter 9

1. Confirm that an equivariant homotopy equivalence between G-spaces induces an isomorphism on equivariant cohomology.

2. Compute the equivariant cohomology of a circle S^1 with $\mathbb{Z}/2\mathbb{Z}$ action given by a reflection.

3. Let X be a G-space with G discrete. Confirm that $H^*_G(X;\mathbb{F}) = \mathrm{Ext}_{\mathbb{F}[G]}(C_*, \mathbb{F})$, where C_* is the singular chain complex of X with the induced structure of an $\mathbb{F}[G]$-module and $\mathbb{F}$ is a trivial $\mathbb{F}[G]$-module.

4. (For people with some knowledge of spectral sequences) Let X be a $\mathbb{Z}/2\mathbb{Z}$-space which is equivariantly homotopy equivalent to a finite-dimensional CW complex. Let $C_* = C_*(X;\mathbb{F}_2)$ be its singular chain complex and τ the induced action on C_*. Confirm that the chain complex
$$C^* \xrightarrow{1+\tau} C^* \xrightarrow{1+\tau} C^* \xrightarrow{1+\tau} \dots$$
has total homology equal to the equivariant homology of X. Prove that $\dim(H^*(X;\mathbb{F}_2)) \geq \dim(H^*(X^{\text{fix}};\mathbb{F}_2)$ using the two spectral sequences associated to this double complex.

5. Give an example of a connected Riemannian manifold with two points p and q such that no geodesic connects p and q.

6. Let G be a compact connected Lie group and let $g \in G$. Show that the centralizer $C_G(g)$ is connected. Show by example that this conclusion fails for noncompact Lie groups.

7. Complete the sequence of exercises on geodesics in homogeneous spaces in Bump 16.4-7 (not reproduced here due to length of notation).