# Math 549: Suggested Exercises for Lectures 1 through 3

References: Teleman Sections 1-15, Gruson and Serganova Chapter 1 and Chapter 2 Sections 1-5.

1. Show that if $G$ is finite, its regular representation is self-dual.
2. Show that the algebra of intertwiners $\mathrm{End}_G(k[G])$ is isomorphic to $k[G]$.
3. Show that the dimension of an irreducible representation of a finite group $G$ is at most its order $|G|$.
4. Classify the irreducible representations of $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$, and $S_3 \times \mathbb{Z}/2\mathbb{Z}$.
5. Write down the character tables of the alternating group $A_4$, the symmetric group $S_4$, and the dihedral group $D_4$. Use the tables to determine the dual of each irreducible representation of these groups.
6. Determine the *structure constants* of the representation rings $Rep(A_4)$ and $Rep(S_4)$ – that is, compute the multiplicities of the irreducible representations $V_k$ in the tensor products $V_i \otimes V_j$. Do this using the character tables and by symmetries, not brute force.
7. Let $A = \mathbb{C}[x, \partial]/(\partial x - x\partial - 1)$ be the ring of polynomial differential operators in one variable. Show that $A$ has no finite-dimensional representations. Give an example of an infinite-dimensional irreducible represenation of $A$.