Math 549: Lie Groups

Updates and edits in response to the university's move to online instruction are included in red.

**Instructor:** Kristen Hendricks

**Email:** kristen.hendricks@rutgers.edu

**Office:** Hill Center 515

**Course Meetings:** MW 1:40-3:00 on Canvas, via BigBlueButton. Please come with questions on the notes, readings, and homework. In the absence thereof I will give an overview of the notes and encourage you to think about specific homework exercises.

**Website:** [www.math.rutgers.edu/~kh754/Math549.html](http://www.math.rutgers.edu/~kh754/Math549.html)

**Office Hours:** MW 4-5, or by appointment on Canvas, via BigBlueButton.

**Prerequisites:** Real Analysis, Linear Algebra, and Elementary Topology, or permission of instructor.

**Assignments:** Suggested homework exercises will be posted weekly.

**Notes:** My scanned lecture notes will be online prior to lecture (intended for help in following along with the lecture, not as a primary reference). Detailed references to our primary readings will be included.

**Topics**

We discussed the following during the first (in-person) half of the course:

- Review of representation theory of finite groups
- The McKay correspondence
- Basics of Lie groups and Lie algebras
- The fundamental group of a Lie group
- The exponential map and the Baker-Campbell-Hausdorff formula (without proof)
- The local and global Frobenius theorems, maximal tori
- The Haar measure, representations of compact Lie groups

During the remaining half of the semester, we will discuss:

- Compact operators and the Peter-Weyl Theorem; the Laplace operator
- Representations of \(\text{sl}(2, \mathbb{C})\)
- Uniqueness up to conjugacy of maximal tori
- Principle bundles
- Introduction to structure theory of Lie algebras – solvable and nilpotent Lie algebras, the universal enveloping algebra
**Primary Resources:**

Kirillov, *Introduction to Lie groups and Lie algebras*
Humphreys, *Introduction to Lie algebras and representation theory*
Bump, *Lie Groups*

**Other Resources:**

**Books (mostly available on SpringerLink)**

Carter, Segal, and MacDonald, *Lectures on Lie groups and Lie algebras*, particularly Chapter 2 by Segal
Brocker and Dieck, *Representations of compact Lie groups*
Fulton and Harris, *Representation theory: a first course*
Knapp, *Lie groups, Lie algebras, and cohomology*
Knapp, *Lie groups beyond an introduction*
Serre, *Complex semisimple Lie algebras*

**Other Online Resources**

Alexandrino and Bettiol, *Introduction to Lie groups, adjoint action and its generalizations*
Gallier, *Notes on Lie group actions: manifolds, Lie groups and Lie algebras*
Sternberg, *Lie Algebras*
Hall, *An Elementary Introduction to Groups and Representations*
Samelson, *Notes on Lie algebras*
Adams, *Root systems and Weyl groups*

**Representation Theory Overview**

Teleman, *Representation theory*
Gruson and Serganova, *A sentimental journey through representation theory*