Math 549: Lie Groups

Updates and edits in response to the university's move to online instruction are included in red.

Instructor: Kristen Hendricks

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Course Meetings: MW 1:40-3:00 on Canvas, via BigBlueButton. Please come with questions on the notes, readings, and homework. In the absence thereof I will give an overview of the notes and encourage you to think about specific homework exercises.

Website: www.math.rutgers.edu/~kh754/Math549.html

Office Hours: MW 4-5, or by appointment on Canvas, via BigBlueButton.

Prerequisites: Real Analysis, Linear Algebra, and Elementary Topology, or permission of instructor.

Assignments: Suggested homework exercises will be posted weekly.

Notes: My scanned lecture notes will be online prior to lecture (intended for help in following along with the lecture, not as a primary reference). Detailed references to our primary readings will be included.

Topics

We discussed the following during the first (in-person) half of the course:

- Review of representation theory of finite groups
- The McKay correspondence
- Basics of Lie groups and Lie algebras
- The fundamental group of a Lie group
- The exponential map and the Baker-Campbell-Hausdorff formula (without proof)
- The local and global Frobenius theorems, maximal tori
- The Haar measure, representations of compact Lie groups

During the remaining half of the semester, we will discuss:

- Compact operators and the Peter-Weyl Theorem; the Laplace operator
- Representations of sl(2, C)
- Uniqueness up to conjugacy of maximal tori
- Principle bundles
- Introduction to structure theory of Lie algebras – solvable and nilpotent Lie algebras, the universal enveloping algebra

Primary Resources:

Kirillov, Introduction to Lie groups and Lie algebras
Humphreys, *Introduction to Lie algebras and representation theory*
Bump, *Lie Groups*

Other Resources:

Books (mostly available on SpringerLink)

Carter, Segal, and MacDonald, *Lectures on Lie groups and Lie algebras*, particularly Chapter 2 by Segal
Brocker and Dieck, *Representations of compact Lie groups*
Fulton and Harris, *Representation theory: a first course*
Knapp, *Lie groups, Lie algebras, and cohomology*
Knapp, *Lie groups beyond an introduction*
Serre, *Complex semisimple Lie algebras*

Other Online Resources

Alexandrino and Bettiol, Introduction to Lie groups, adjoint action and its generalizations
Gallier, Notes on Lie group actions: manifolds, Lie groups and Lie algebras
Sternberg, Lie Algebras
Hall, An Elementary Introduction to Groups and Representations
Samelson, Notes on Lie algebras
Adams, Root systems and Weyl groups

Representation Theory Overview

Teleman, Representation theory
Gruson and Serganova, A sentimental journey through representation theory