Math 541: Introduction to Algebraic Topology II

Instructor: Kristen Hendricks

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Course Location and Time: MW 2:00-3:20 in HLL-525

Website: www.math.rutgers.edu/~kh754/Math541.html, or equivalently Canvas.

Office Hours: Monday shortly after class at department coffee hour (similarly Wednesday if there is not a colloquium), or by appointment.

Additional Schedule Logistics: I will be out of town the week of March 27. We will jointly find a time to reschedule the two lectures from that week.

Masking: I intend to mask for lecture if any students are doing so. Obviously, coffee hour is unmasked; if you would like a masked or online office hour, let me know and we will set one up.

Prerequisites: Having read and understood the first three chapters of Hatcher (or equivalent) excluding the special topics.

Topics: This is a second course in algebraic topology. We will cover homotopy theory, characteristic classes, and possibly some spectral sequences if time allows.

Assignments: Homework exercises for each week will be posted some time before the start of Monday's lecture, and distributed in class. Registered students are expected to write up and hand in roughly half the homework exercises sometime before the end of term. Exercises may be handed in via hard copy or (mildly preferred) canvas upload.

Notes: Lecture notes will typically be posted a few hours before lecture, and updated afterward to with any corrections or changes to the amount of material covered in class.

Texts: Reading will be announced in advance of the lecture on the homework sheets. Texts will be as follows:

Homotopy Theory: A. Hatcher *Algebraic Topology*; J. Milnor, *Topology from the Differentiable Viewpoint*
Characteristic Classes: J. Milnor and W. Stasheff, *Characteristic Classes*; A. Hatcher *Vector Bundles and K-Theory*
Spectral Sequences (time permitting): A. Hatcher, *Spectral Sequences*; R. Bott and L. Tu, *Differential Forms in Algebraic Topology*

Motivation: Here are a few questions that can be either addressed or helpfully rephrased using the techniques of this course:

Given a closed manifold M, a framed submanifold N is an embedded submanifold together with a smoothly varying basis for the normal bundle at each point in the set. What can one say about the set of such submanifolds, up to framed cobordism in M? How does this change if the framing condition is removed?

Given a smooth manifold M of dimension m, what are the smallest numbers n and k such that M may be immersed into n-dimensional Euclidean space and embedded into k-dimensional Euclidean space?

For what values of n is there a bilinear multiplication on R^n without zero divisors? We are aware of n=1 (the real numbers), n=2 (the complex numbers), n=4 (the quaternions), and n=8 (the octonions). Are there any others?

How can one distinguish between smooth manifolds that are homeomorphic but not diffeomorphic?