Math 540: Exercises for Week 7

Reading: Hatcher Section 2.1

1. Hatcher 2.1.1

2. Hatcher 2.1.3 (Also, compute the homology of the Δ-complex you construct in this exercise.)

3. Hatcher 2.1.5

4. Hatcher 2.1.8

5. Hatcher 2.1.11

6. Hatcher 2.1.12

7. Hatcher 2.1.13

8. Construct explicit chain homotopy equivalences between the following pairs of chain complexes.

   (a) The chain complex

   $$0 \to \mathbb{Z}\langle a, b \rangle \xrightarrow{\partial_1} \mathbb{Z}\langle v, w \rangle \to 0$$

   where the map ∂_1 takes $\partial_1(a) = \partial_1(b) = v - w$ and the chain complex

   $$0 \to \mathbb{Z}\langle c \rangle \xrightarrow{\partial_1} \mathbb{Z}\langle y \rangle \to 0$$

   with $\partial_1 \equiv 0$.

   (b) The chain complex

   $$0 \to \mathbb{Z}\langle U, L \rangle \xrightarrow{\partial_2} \mathbb{Z}\langle a, b, c \rangle \xrightarrow{\partial_1} \mathbb{Z}\langle v \rangle \to 0$$

   where $\partial_2(U) = \partial_2(L) = a + b - c$ and $\partial_1 \equiv 0$ and the complex

   $$0 \to \mathbb{Z}\langle E \rangle \xrightarrow{\partial_2} \mathbb{Z}\langle f, g \rangle \xrightarrow{\partial_1} \mathbb{Z}\langle w \rangle \to 0$$

   with all differentials identically zero.