Math 540: Exercises for Week 4

Reading: Hatcher Section 1.2-3

1. Use Van Kampen's Theorem to compute the fundamental group of the Klein bottle. Find a two-sheeted covering map from the torus to the Klein bottle, and compute the induced map of fundamental groups.

2. Consider the space X built from a seven-sided polygonal region by means of the labelling scheme $abaaab^{-1}a^{-1}$. Show that the fundamental group of X is the free product of two cyclic groups.

3. Hatcher 1.2.3

4. Hatcher 1.2.9

5. Hatcher 1.2.10

6. Hatcher 1.2.22 [Note: The explanation in this exercise may be a little dense; feel free to drop by office hours for some picture-drawing.]

7. (Included as an optional exercise because it came up in office hours) Let G be a path-connected topological group. Let $f \colon I \to G$ and $g \colon I \to G$ be two loops in G based at the identity. Observe that there are two natural notions of the product of f and g: the composition $f \cdot g$ defined in class, and the pointwise product $fg(s) = f(s)g(s)$. Prove that these notions agree up to homotopy of loops. Use this to show the fundamental group of a topological group is always abelian.