Math 540: Exercises for Week 3

Reading: Hatcher Section 1.1-2

1. Hatcher 1.1.9

2. Hatcher 1.1.10

3. Hatcher 1.1.16 [If you're doing a proper subset of the exercises this week, make sure to do this one.]

4. Hatcher 1.1.17

5. Hatcher 1.1.18

6. Let $H^1(X) = [X, S^1]$ be the set of homotopy classes of continuous maps from X to the circle. (No basepoints involved.)

    (a) Remember that S^1 has the structure of a topological group. Use this to give $H^1(X)$ the structure of an abelian group.

    (b) What is $H^1(\{\text{pt}\})$?

    (c) What is $H^1(S^1)$? (Use what you know about $\pi_1(S^1)$.)

    (d) Show that H^1 is functorial in the following sense: Given $f \colon X \to Y$ a continuous map, there is an induced map $f^* \colon H^1(Y) \to H^1(X)$. Moreover if $g \colon Y \to Z$ then $(g \circ f)^* = f^* \circ g^*$ as maps $H^1(Z) \to H^1(X)$.

    (e) Show that if f and h are homotopic as maps $X \to Y$ then $f^* = h^*$. Conclude that if $X \simeq Y$ then $H^1(X) \simeq H^1(Y)$.

    (f) Use H^1 to prove there is no retraction $D^2 \to S^1$, giving yet another proof of the Brouwer fixed point theorem.