Math 540: Exercises for Week 14

Reading: Hatcher Section 3.3

Remark: The last few exercises here are meant to be challenge exercises for the enthusiastic.

1. Hatcher 3.3.17
2. Hatcher 3.3.20
3. Hatcher 3.3.21
4. Hatcher 3.3.24 [This example is particularly important in low-dimensional topology.]
5. Read Theorem 3.43 and do Hatcher 3.32 and 3.33.
6. Recall that a knot K is a smooth embedding of S^1 into S^3. For a knot K, consider the manifold $X_K = S^3 - \nu(K)$ obtained by deleting a neighborhood of K, so that X_K is a closed manifold with torus boundary.

   (a) What is $H_*(X_K; \mathbb{Z})$? [Note: We computed π_1 of this manifold in the exercises to Chapter 1, but you don't need that to do this computation.]

   (b) The boundary torus of X_K has two distinguished homology classes of curves μ and λ, defined as follows. The curve μ is a curve such that μ bounds a disk in $\overline{\nu(K)}$ which intersects K exactly once, and the curve λ is the unique up to homotopy curve on the torus which intersects μ once and is unlinked with K. Let H be a solid torus with T its boundary torus, and let $S^3_{p/q}(K) = X_K \coprod_T H$ identified along a map which carries a meriodional curve of H on T to a curve in the homology class $p\mu + q\lambda$ on ∂X_K. This is called the p/q surgery on K. What is the homology of $S^3_{p/q}(K)$?