Math 540: Exercises for Week 7

Reading: Hatcher Section 2.1

Note that this is not as long as it looks; some of the exercises are quick.

1. Hatcher 2.1.1

2. Hatcher 2.1.3

3. Hatcher 2.1.5

4. Hatcher 2.1.8

5. Hatcher 2.1.11

6. Hatcher 2.1.12

7. Construct explicit chain homotopy equivalences between the following pairs of chain complexes.

   (a) The chain complex

   $$0 \to \mathbb{Z}\langle a, b \rangle \xrightarrow{\partial_1} \mathbb{Z}\langle v, w \rangle \to 0$$

   where the map ∂_1 takes $\partial_1(a) = \partial_1(b) = v - w$ and the chain complex

   $$0 \to \mathbb{Z}\langle c \rangle \xrightarrow{\partial_1} \mathbb{Z}\langle y \rangle \to 0$$

   with $\partial_1 \equiv 0$.

   (b) The chain complex

   $$0 \to \mathbb{Z}\langle U, L \rangle \xrightarrow{\partial_2} \mathbb{Z}\langle a, b, c \rangle \xrightarrow{\partial_1} \mathbb{Z}\langle v \rangle \to 0$$

   where $\partial_2(U) = \partial_2(L) = a + b - c$ and $\partial_1 \equiv 0$ and the complex

   $$0 \to \mathbb{Z}\langle E \rangle \xrightarrow{\partial_2} \mathbb{Z}\langle f, g \rangle \xrightarrow{\partial_1} \mathbb{Z}\langle w \rangle \to 0$$

   with all differentials identically zero.