

Math 540: Exercises for Week 10

Reading: Hatcher Section 2.2, 2.3

Note that some of the exercises below are quite short.

1. Hatcher 2.2.28
2. Hatcher 2.2.29 [This is an important example.]
3. Hatcher 2.2.31-32
4. Hatcher 2.2.35 [Also particularly recommended.]
5. Hatcher 2.2.38 [We'll briefly use this algebra lemma at the end of Wednesday's lecture.]
6. Hatcher 2.2.41
7. Hatcher 2.3.2 [Ties in strongly to what we'll do next chapter.]
8. Let S^3 be represented by the unit sphere in \mathbb{C}^2 . For p and q coprime nonzero integers, the lens space $L(p, q)$ is the quotient of S^3 by the action of $\mathbb{Z}/p\mathbb{Z}$ generated by

$$(z_1, z_2) \mapsto \left(e^{\frac{2\pi i}{p}}(z_1), e^{\frac{2q\pi i}{p}}(z_2) \right).$$

So for example, $\mathbb{R}\mathbb{P}^3$ is $L(2, 1)$. Use the quotient of the decomposition of S^3 into two solid tori (which is preserved by this group action) and the Mayer-Vietoris sequence to compute the homology of $L(p, q)$.

Note that we might start but probably not finish this example in class. Note also that Hatcher's notation for general lens spaces is not quite identical to the above.