

Math 411: Homework 1

Due Monday September 16 at 5 pm.

Upcoming office hours: Thursday September 5 10-11 in LSH B-106, Monday September 9 1-2 on Zoom, Thursday September 12 10-11 in LSH B-106.

Reading: Supplementary Readings on Canvas (some of which will be review from previous courses); Rudin 1.1-18.

1. Send me an email introducing yourself. Let me know what your favorite result from Math 311 (or equivalent) was, and anything else you think I should know about your background.
2. Do exercises 15, 20, 28, 29, 33(a),(c), and 34 from the “Sets and Functions” supplementary reading.
3. Do exercises 3 and 9 from the “Equivalence Relations” supplement.
4. Prove by induction that $\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$.
5. Let \sim be an equivalence relation on a set S . Prove that if $[a]$ and $[b]$ are two equivalence classes, either $[a] = [b]$ or $[a] \cap [b] = \emptyset$.
6. Prove using the Peano axioms and the definitions of $+$ and \times on \mathbb{N} given in class that for $a, b \in \mathbb{N}$, $a \times b = b \times a$. [You can use that we already checked that $a \times 1 = 1 \times a$.]
7. Suppose that A and B are sets, and denote by A^B the set of all maps $f: B \rightarrow A$. Construct a bijection between $A^{B \times C}$ and $(A^B)^C$.
8. Prove that if A and B are nonempty finite sets, then $|A^B| = |A|^{|B|}$.

Remark 1: You have an unusual amount of time to do this assignment and some of its exercises are short; correspondingly, it is slightly longer than will be the norm.

Remark 2: It is possible that we will not have covered quite enough material for all of these questions by the end of lecture Monday September 9. Should that occur, the affected questions will be moved to Homework 2.