Math 354, Section 04
Linear Optimization

Quiz

Instructions: You have 40 minutes to complete the quiz. There are seven questions worth a total of eighteen points. Partial credit will be given for progress toward correct solutions where relevant. You may not use any books, notes, calculators, or other electronic devices.

Name: _____

RUID: _____

Question	Points	Score
1	3	
2	3	
3	3	
4	3	
5	2	
6	4	
Total:	18	

Consider the following linear programming problem.

A taxi company wishes to transport the largest possible number of people from the New Brunswick train station to Livingston Campus. The company can run cars, which will transport five people, and vans, which will transport eight people. There are ten drivers available total, and the company wishes to have at least five vehicles on the road at all times. A car uses \$2 worth of gas to make the trip and a van uses \$4 worth of gas to make the trip. A car uses \$2 worth of gas to make the trip and a van uses \$4 worth of gas to make the trip; the company limits gas expenses to \$24 a day. What number of cars and vans will allow the taxi company to transport the maximum number of people possible?

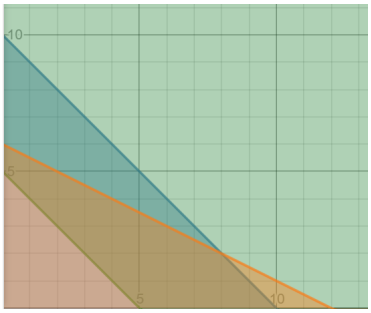
1. [3pts.] Write down this problem as a system of equations in standard form, including both the objective functions and the constraints.

Solution: Let x_1 be the number of cars and x_2 be the number of vans. We wish to maximize $z = 5x_1 + 8x_2$ subject to the constraints

$$\begin{cases} x_1 + x_2 \leq 10 \\ -x_1 - x_2 \leq -5 \\ 2x_1 + 4x_2 \leq 24 \\ x_1, x_2 \geq 0 \end{cases}.$$

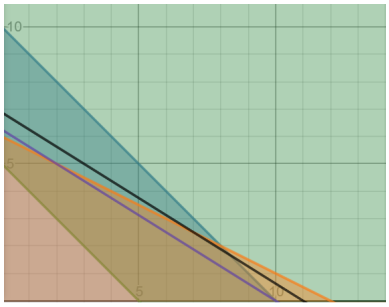
2. [3pts.] Graph the set of feasible solutions to the problem.

Solution: The region of feasible solutions is the triply-shaded region shown.



3. [3pts.] Find the optimal number of cars and vans. Justify your answer.

Solution: We draw a few level sets of the line z=5x+8y as shown and conclude the optimal solution is at (8,2); which is to say, eight cars and two vans, transporting a total of 56 people.



4. [3pts.] Transform your set of equations from question (2) into canonical form. Write this down both as a system of equations and in matrix notation.

Solution: In equations, we wish to maximize $z = 5x_1 + 8x_2$ subject to

$$\begin{cases} x_1 + x_2 + u_1 = 10 \\ -x_1 - x_2 + u_2 = -5 \\ 2x_1 + 4x_2 + u_3 = 24 \\ x_1, x_2, u_1, u_2, u_3 \geq 0 \end{cases}$$

Equivalently in matrix form we would like to maximize

$$z = \begin{bmatrix} 5 & 8 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

subject to the constraints

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 \\ 2 & 4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ 24 \end{bmatrix}, \qquad \begin{bmatrix} x_1 \\ x_2 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

5. [2pts.] What are the values of the slack variables from problem (4) at the optimal solution you found in problem (3)? What does this mean physically?

Solution: The values of the slack variables are $u_1 = 0$, representing that all ten drivers are busy, $u_2 = 5$, representing that there are zero more total vehicles on the road than the company's minimum, and $u_3 = 0$, representing that the entire fuel budget is used with no surplus.

6. [4pts.] Suppose the company acquires a new set of vans capable of transporting ten people at the same fuel cost. What combination of cars and vans results in the maximum number of people being transported now? In this situation, if you were running the taxi company, what would you actually choose to do?

Solution: We draw a few level sets of $z = 5x_1 + 10x_2$ as shown and conclude that the maximum number of people transported happens anywhere along the line segment between (8,2) and (0,6) – 60 people at all points along the line. In this situation we would probably actually run six vans and no cars, since in real life employing the drivers probably also costs money and wear and tear on vehicles is a consideration. (Any other reasonable comment here would be accepted, the point of the question is to think about what other considerations might be in play.)