

Math 354, Section 04
Linear Optimization
Sample Midterm 2

Instructions: You have 80 minutes to complete the exam. There are three questions, worth a total of 40 points. Partial credit will be given for progress toward correct solutions where relevant. Solutions will be graded on their correctness relative to previous steps where this makes sense. You may not use any books, notes, calculators, or other electronic devices.

Name: _____

Question	Points	Score
1	20	
2	10	
3	10	
Total:	40	

1. Consider the following linear programming problem.

A farmer owns a farm that produces corn, soybeans, and oats. She has 12 acres of land available for cultivation. Planting requires different amounts of money for seed and different amounts of total labor. (After planting the requirements for cultivating and harvesting are sufficiently similar that they can be ignored for purposes of optimization.) Corn costs a total of \$36 per acre to plant and needs 6 hours of labor per acre; the farmer expects corn to yield a net profit of \$40 per acre. Soybeans cost a total of \$24 per acre to plant and need 6 hours of labor per acre; the farmer expects soybeans to yield a net profit of \$30 per acre. Oats cost \$18 per acre to plant and need 2 hours of labor per acre; the farmer expects oats to yield a net profit of \$20 per acre. The farmer has 48 total labor-hours available for planting and \$360 available for initial costs. How much of each crop should be planted to maximize profit, and what is the maximum profit thusly obtained?

- (a) [2pts.] Write down the situation above as a linear programming problem in standard form.
- (b) [6pts.] Solve this problem using the simplex method, and determine the optimal solution and optimal value.
- (c) [5pts.] What is the dual problem to the primal problem from part (a)? What are its optimal solution and optimal value? What do the entries of the optimal solution mean?
- (d) [2pts.] The farmer becomes worried that she might be wrong about what the net profit on soybeans will be at harvest time. What is the range of values the profit on soybeans could be such that the solution you found in part (b) is still optimal?
- (e) [2pts.] The farmer becomes worried that she might be wrong about the net profit on corn will be at harvest time. What is the range of values the profit on corn could be such that the solution you found in part (b) is still optimal?
- (f) [3pts.] Unfortunately the farmer turns out to only have 42 hours of labor available! What should she do in this situation to maximize the profit she can get from her farm, and what is her new maximum profit?

This page is for your solution to Problem 1.

This page is also for your solution to Problem 1.

2. Consider the following linear programming problem: Maximize $z = 3x_1 - 2x_2$ subject to

$$\begin{cases} x_1 + x_2 + 2x_3 \leq 7 \\ 2x_1 + x_2 + x_3 = 4 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

- (a) [5pts.] Solve the problem using the two-phase method.
- (b) [5pts.] Write down the dual to the problem above. What is its optimal solution and optimal value?

3. Consider the following simplex tableaux.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	z	
x_4	0	0	2	1	$5/2$	0	0	0	5
x_1	1	0	5	0	-3	0	-2	0	3
x_6	0	0	3	0	4	1	-4	0	1
x_2	0	1	0	0	$3/2$	0	0	0	1
	0	0	-2	0	0	0	-6	1	62

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	z	
x_4	0	0	2	1	$5/2$	0	0	0	6
x_1	1	0	5	0	-3	0	-2	0	-4
x_6	0	0	-2	0	4	1	-4	0	-2
x_2	0	1	0	0	$3/2$	0	0	0	1
	0	0	3	0	2	0	5	1	-80

- [2pts.] For the first tableau, what basic feasible solution does the tableau represent?
- [2pts.] For the first tableau, what is the correct entering variable if the standard rule for the simplex method is used, and in that case what is the departing variable?
- [3pts.] For the first tableau, what is the correct entering variable if Bland's Rule for the simplex method is used, and in that case what is the departing variable?
- [3pts.] For the second tableau, suppose I want to do a dual pivot toward making the tableau feasible. What are the correct entering and departing variables?

This page is for scratch work. If you want anything on it graded, indicate that this is the case **very clearly** on the original problem page.

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