

Math 354, Section 04
Linear Optimization
Midterm 2

Instructions: You have 80 minutes to complete the exam. There are four questions, worth a total of 40 points. Partial credit will be given for progress toward correct solutions where relevant. You may not use any books, notes, calculators, or other electronic devices.

Name: _____

Question	Points	Score
1	20	
2	10	
3	10	
Total:	40	

1. Consider the following linear programming problem.

A lawn products company has available 100 tons of nitrate and 60 tons of phosphate to use in producing its three types of fertilizer for sale this week. Their three fertilizer types are regular lawn, super lawn, and garden. Every thousand bags of regular lawn fertilizer requires 4 tons of nitrate and 2 tons of phosphate, and can be sold for a profit of \$300. Every thousand bags of super lawn fertilizer require 3 tons of nitrate and 3 tons of phosphate, and can be sold for a profit of \$500. Every thousand bags of garden fertilizer requires 3 tons of nitrate and 2 tons of phosphate and can be sold for a profit of \$400. Fractional numbers of bags of fertilizer are ok and can be retailed appropriately.

- (a) [2pts.] Write this situation down as a linear programming problem in standard form.
- (b) [5pts.] Solve the linear programming problem using the simplex method.
- (c) [2pts.] The lawn company has the option of buying another ton of nitrate to increase production this week. Should it do that? If so, what is the maximum price it should pay for the nitrate?
- (d) [2pts.] The lawn company has the option of selling off a ton of phosphate this week. What is the minimum price it should accept for the phosphate?
- (e) [3pts.] The lawn products company expects that next week, because most people will have planted their gardens already, it will have to sell garden fertilizer at a discount. By how much can it lower the price of the garden fertilizer per thousand bags such that the solution you found in part (b) remains optimal?
- (f) [3pts.] The lawn products company expects that next week many people will want to encourage their lawns to grow, and the price at which it could sell super lawn fertilizer may increase. How much does it need to increase before the company's best strategy changes?
- (g) [3pts.] Due to a miscommunication only 80 tons of nitrate have been delivered to the company factory! What should the company do under this circumstance, and what is the new maximum profit for the week?

This page is for your solution to Problem 1.

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2. Consider the following linear programming problem: maximize $z = 3x_1 - 2x_2 + x_4$ subject to

$$\begin{cases} x_1 + x_3 + x_4 \leq 10 \\ 2x_2 - 6x_3 \geq 8 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

- (a) [5pts.] Solve the problem using the two-phase method.
- (b) [5pts.] What is the dual of this linear programming problem? What are the optimal solution and optimal value of the dual problem?

3. Consider the following simplex tableaux.

	x_1	x_2	x_3	x_4	x_5	u_1	u_2	z	
x_5	2	-2	0	0	1	0	0	0	2
x_3	-3	3	1	0	0	0	2	0	0
u_1	-4	0	0	0	0	1	4	0	7
x_4	1	1	0	1	0	0	0	0	3
	-1	3	0	0	0	0	-7	1	-5

	x_1	x_2	x_3	x_4	x_5	u_1	u_2	z	
x_5	-2	-2	0	0	1	0	0	0	-12
x_3	-3	3	1	0	0	0	2	0	2
u_1	-4	0	0	0	0	1	-2	0	-9
x_4	1	1	0	1	0	0	0	0	3
	4	3	0	0	0	0	7	1	62

- [2pts.] For the first tableau, what basic feasible solution does the tableau represent?
- [2pts.] For the first tableau, what is the correct entering variable if the standard rule for the simplex method is used, and in that case what is the departing variable?
- [3pts.] Suppose that my attempt to run the simplex rule using the standard rule on the first tableau seems to be cycling. To fix this, what choice of entering and departing variable should I make?
- [3pts.] For the second tableau, suppose I want to do a dual pivot toward making the tableau feasible. What entering and departing variables should I use?

This page is for scratch work. If you want anything on it graded, indicate that this is the case **very clearly** on the original problem page.

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