Math 354, Section 04
Linear Optimization

Sample Midterm 1

Instructions: You have 80 minutes to complete the midterm. There are four questions, worth a total of 40 points. Partial credit will be given for progress toward correct solutions where relevant. You may not use any books, notes, calculators, or other electronic devices.

Name: ____________________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. For each of the following sets of vectors, determine whether it is linearly independent and find the dimension of its span.

   (a) [3pts.]

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -12 \\ 5 \\ 4 \\ 18 \end{bmatrix} \right\} \subseteq \mathbb{R}^4$$

   (b) [3pts.]

$$\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -9 \\ -4 \\ 1 \end{bmatrix} \right\} \subseteq \mathbb{R}^3$$

   (c) [4pts.]

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\} \subseteq \mathbb{R}^5$$

2. Consider the following linear programming problem in canonical form: maximize $z = 3x_2 - x_5$ subject to the constraints

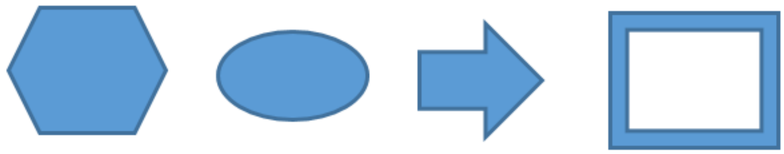
$$\begin{cases} x_1 + 2x_2 = 6 \\ 2x_2 + x_3 + 3x_5 = 7 \\ 2x_3 + x_4 - x_5 = 0 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

   (a) [7pts.] Find all of the basic solutions to the problem. Which of them are basic feasible?

   (b) [3pts.] Assuming that the set of feasible solutions is bounded, what is the maximum value of z subject to the constraints?

3. (a) [6pts.] For each of the following subsets of the plane, decide whether it is convex and whether it is bounded. If if is convex, say what the extreme points are. No need to justify your answers.

   - Each of the four subsets of the plane shown below.



   - The closed half-space $H = \{\mathbf{x} : 3x_1 + 2x_2 \leq 6\}$.
   - The set of feasible solutions to

$$\begin{cases} -x_1 + -x_2 \leq -1 \\ x_1, x_2 \geq 0 \end{cases}$$

   (b) [4pts.] In class we showed that the intersection of two convex subsets A and B in $\mathbb{R}^n$ is necessarily convex. (Recall that the intersection of A and B is the set of points that appear in both A and B, which visually looks like their overlap.) The *union* of two sets A and B is the set of all points appearing in either A or B. If A and B are convex, is their union necessarily convex? Support your answer.

4. A snack company makes chili-flavored and pizza-flavored potato chips. The chips must go through three processes: frying, flavoring, and packing. A kilogram of chili-flavored chips takes 3 minutes to fry, 3 minutes to flavor, and 3 minutes to pack. A kilogram of pizza-flavored chips takes 3 minutes to fry, 4 minutes to flavor, and 2 minutes to pack. The net profit on a kilogram of chili-flavored chips is 10 cents and the net profit on a kilogram of pizza-flavored chips is 12 cents. The fryer is available for 4 hours every day, the flavorer is available for 4 hours every day, and the packer is available for 3 hours every day.
   (a) [2pts.] Put this linear programming problem into a set of equations in standard form.
   (b) [2pts.] Sketch the region of feasible solutions to your standard form model.
   (c) [2pts.] Transform your equations from part (a) into canonical form.
   (d) [2pts.] Use your sketch to identify all of the basic feasible solutions to this linear programming problem. [Don't try to find all the basic solutions, there are quite a few.]
   (e) [2pts.] How many chips of each kind should be made to maximize the bakery's profit?

This page is for scratch work. If you want anything on it graded, indicate that this is the case **very clearly** on the original problem page.