Math 354, Section 04
Linear Optimization

Midterm 1

Instructions: You have 80 minutes to complete the exam. There are four questions, worth a total of 40 points. Partial credit will be given for progress toward correct solutions where relevant. You may not use any books, notes, calculators, or other electronic devices.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. For each of the following sets of vectors, determine whether it is linearly independent and find the dimension of its span.

   (a) [3pts.]

$$\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 7 \end{bmatrix} \right\} \subseteq \mathbb{R}^4$$

   (b) [3pts.]

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\} \subseteq \mathbb{R}^3$$

   (c) [4pts.]

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \\ 1 \end{bmatrix} \right\} \subseteq \mathbb{R}^5$$

2. Consider the following linear programming problem in canonical form: maximize $z = x_1 - x_2 + x_3$ subject to the constraints

$$\begin{cases} 2x_1 + x_2 + x_3 + 7x_4 = 7 \\ 2x_2 - 2x_4 = 4 \\ x_1 + 4x_2 + x_3 = 12 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

   (a) [7pts.] Find all of the basic solutions to the problem. Which of them are basic feasible?

   (b) [3pts.] Assuming that the set of feasible solutions is bounded, what is the maximum value of z subject to the constraints?

3. For each of the following, draw an example of a set with the described properties. Your answers can be subsets of $\mathbb{R}$, of $\mathbb{R}^2$, or of $\mathbb{R}^3$ as you find appropriate.

   (a) [2pts.] A convex set with five extreme points.

   (b) [2pts.] A convex set with no extreme points.

   (c) [2pts.] A convex set with infinitely many extreme points.

   (d) [2pts.] An unbounded convex set with exactly one extreme point.

   (e) [2pts.] A bounded convex set with exactly one extreme point.

4. A bakery makes two kinds of doughnuts: glazed and dipped in powdered sugar. It makes a profit of 7 cents on each glazed doughnut sold and a profit of 5 cents on each powdered sugar doughnut sold. There are three main operations in doughnut making: baking (necessary for both kinds of doughnuts), glazing (necessary for the glazed doughnuts), and dipping (necessary for the powdered sugar doughnuts). The bakery's kitchen has the capacity to bake at most 1400 doughnuts, glaze at most 1000 doughnuts, and dip at most 1200 doughnuts every day. Because glazed doughnuts are very popular and important to the reputation of the store, the manager of the store has said that at least 600 should be made every day. What combination of doughnut types should the store make to maximize its profit?

   (a) [2pts.] Put this linear programming problem into a set of equations in standard form.

   (b) [2pts.] Sketch the region of feasible solutions to your standard form model.

   (c) [2pts.] Transform your equations from part (a) into canonical form.

   (d) [2pts.] Use your sketch to identify all of the basic feasible solutions to this linear programming problem. [Don't try to find all the basic solutions, there are quite a few.]

   (e) [2pts.] How many doughnuts of each kind should be made to maximize the bakery's profit?

This page is for scratch work. If you want anything on it graded, indicate that this is the case **very clearly** on the original problem page.

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