Homework 6 Solutions

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Problem 4

We are interested in the matrix equation

$$\begin{bmatrix} 2 & 2 & 3 & 1 & 0 \\ 4 & 6 & 10 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 420 \\ 600 \end{bmatrix}$$

which has the basic feasible solutions

$$\mathbf{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 420 \\ 600 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 150 \\ 0 \\ 0 \\ 120 \\ 0 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 0 \\ 100 \\ 0 \\ 220 \\ 0 \end{bmatrix}, \mathbf{z} = \begin{bmatrix} 0 \\ 0 \\ 60 \\ 240 \\ 0 \end{bmatrix}$$

In all cases the basic variables of each basic feasible solution are exactly the variables which correspond to nonzero entries (that is, there are no degenerate solutions) so for $\mathbf{v}$ the basic variables are $\{u_1, u_2\}$, for $\mathbf{w}$ they are $\{x_1, u_1\}$, for $\mathbf{x}$ they are $\{x_2, u_1\}$ and for $\mathbf{z}$ they are $\{x_3, u_1\}$. This means that every pair of basic feasible solutions are adjacent to each other (each pair shares all but one basic variable, namely u_1, and differs in the remaining basic variable). The shape is an irregular tetrahedron.

Problem 5

We recall that we are interested in the matrix equation

$$\begin{bmatrix} 3 & 3 & 1 & 0 & 0 \\ 3 & 4 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 240 \\ 240 \\ 180 \end{bmatrix}$$

which has basic feasible solutions

$$\mathbf{v} = \begin{bmatrix} 0 \\ 0 \\ 240 \\ 240 \\ 180 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 60 \\ 0 \\ 60 \\ 60 \\ 0 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 0 \\ 60 \\ 60 \\ 0 \\ 60 \end{bmatrix}, \mathbf{z} = \begin{bmatrix} 40 \\ 30 \\ 30 \\ 0 \\ 0 \end{bmatrix}$$

In all cases the basic variables of each basic feasible solution are exactly the variables which correspond to nonzero entries (that is, there are no degenerate solutions) so for $\mathbf{v}$ the basic variables are $\{u_1, u_2, u_3\}$, for $\mathbf{w}$ they are $\{x_1, u_1, u_2\}$, for $\mathbf{x}$ they are $\{x_2, u_1, u_3\}$ and for $\mathbf{z}$ they are $\{x_1, x_2, x_3\}$. This means that $\mathbf{v}$ is adjacent to $\mathbf{w}$ and $\mathbf{x}$, while $\mathbf{w}$ is adjacent to $\mathbf{v}$ and $\mathbf{z}$, whereas $\mathbf{x}$ is adjacent to $\mathbf{v}$ and $\mathbf{z}$, and finally symmetrically $\mathbf{z}$ is adjacent to $\mathbf{w}$ and $\mathbf{x}$. The shape is a quadrilateral.

Problem 6

We recall that the canonical form of this problem is to maximize $z = .5x_1 + .8x_2 + 1.2x_3$ subject to the constraints

$$\begin{bmatrix} 2 & 2 & 3 & 1 & 0 \\ 4 & 6 & 10 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 420 \\ 600 \end{bmatrix}, \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ u_1 \\ u_2 \end{bmatrix} \geq 0.$$

This becomes the initial tableau shown below.

	x_1	x_2	x_3	u_1	u_2	z	
u_1	2	2	3	1	0	0	420
u_2	4	6	10	0	1	0	600
	$-.5$	$-.8$	-1.2	0	0	1	0

Problem 7

We recall that the canonical form of this problem is to mazimize $z = 10x_1 + 12x_2$ subject to the constraints

$$\begin{bmatrix} 3 & 3 & 1 & 0 & 0 \\ 3 & 4 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 240 \\ 240 \\ 180 \end{bmatrix}, \qquad \begin{bmatrix} x_1 \\ x_2 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} \geq \mathbf{0}.$$

This becomes the initial tableau shown below.

	x_1	x_2	u_1	u_2	u_3	z	
u_1	3	3	1	0	0	0	240
u_2	3	4	0	1	0	0	240
u_3	3	2	0	0	1	0	180
	-10	-12	0	0	0	1	0