MATH 354: Homework 5

Due: February 24, 2022 at 11:00 am

1. Upcoming office hours are Monday February 21 3-4 pm, and Thursday February 24 9-10 am.

2. Reminder that Midterm 1 is coming up on Monday February 28 in class. It will cover the material of lecture up to the start of the simplex method (so sometime during lecture Monday February 21, depending on how many loose ends about basic solutions we need to wrap up first). A sample exam will be posted sometime early next week; your TA will schedule a review session to discuss the problems with you, which will be announced on Canvas.

3. Read Sections 7.1-4 in Miller, or equivalently Section 2.1 in Kolman and Beck.

4. Put Problem 6 from Homework 1 into canonical form. Then find the basic solutions and basic feasible solutions. What is the maximum profit the book publisher can make, and how many books of each type should be produced for that max profit?

5. Put Problem 7 from Homework 1 into canonical form. Find three different basic solutions to the problem. [Note: There are more than that in total, don't try to find all of them.]

6. Find the basic and basic feasible solutions of a linear programming problem whose constraints are
$$\begin{bmatrix} 1 & 0 & 2 & -1 \\ 2 & 1 & 4 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 1 \end{bmatrix}, \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \mathbf{0}.$$
The region described by these constraints is bounded (you don't have to prove that, but if you're bored, satisfy yourself that it's true). Given that fact, what is the maximum value of $z = 2x_1 + x_3$ on the region?

7. Consider the linear programming problem whose constraints are
$$\begin{bmatrix} 2 & 3 & 1 & 0 & 0 \\ -1 & 1 & 0 & 2 & 1 \\ 0 & 6 & 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \geq \mathbf{0}.$$
Which of the following vectors are basic solutions? Which are basic feasible?
$$\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0 \\ 2 \\ -5 \\ 0 \\ -1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{z} = \begin{bmatrix} 0 \\ .25 \\ .25 \\ 0 \\ .75 \end{bmatrix}$$