MATH 354: Homework 4

Due: February 17, 2022 at 11:00 am

1. Upcoming office hours are Monday February 14 3-4 pm, and Thursday February 17 9-10 am.

2. Read Sections 6.1-4 in Miller, or equivalently Section 1.5 in Kolman and Beck.

3. Write down a linear programming problem in standard form using at least two variables with an unbounded set of feasible solutions and no optimal solution.

4. Let $S \subseteq \mathbb{R}^n$ be a bounded set contained in some rectangle $R = \{\mathbf{x} : -M_j \leq x_j \leq M_j\}$ for some positive real numbers M_j for $j = 1, \ldots n$. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a linear map. Prove that the image $f(S)$ is a bounded subset of $\mathbb{R}^m$. [Hint: Represent the linear map by multiplication by an $m \times n$ matrix $\mathbf{D}$, so that $f(\mathbf{x}) = \mathbf{D}\mathbf{x}$. What is the ith coordinate of $f(\mathbf{x})$?]

5. Give an example of a continuous function $f : \mathbb{R} \to \mathbb{R}$ and a bounded set S such that $f(S)$ is not bounded. [Hint: Don't overthink this.]

6. Decide whether each of the following sets of vectors is linearly independent. In each case, what is the dimension of the span of the vectors?

   (a)
   $$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} \right\} \subset \mathbb{R}^3$$

   (b)
   $$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 14 \end{bmatrix} \right\} \subseteq \mathbb{R}^2$$

   (c)
   $$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\} \subseteq \mathbb{R}^3$$

   (d)
   $$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\} \subseteq \mathbb{R}^2$$

   (e)
   $$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -7 \end{bmatrix} \right\} \subseteq \mathbb{R}^3$$

7. Let
   $$\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \qquad \mathbf{v_2} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

   What is the set of vectors $\mathbf{v_3}$ such that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ forms a linearly independent set in $\mathbb{R}^3$?