

MATH 354: Homework 3

Due: February 10, 2022 at 11:00 am

1. Upcoming office hours are Monday February 7 3-4 pm, and Thursday February 10 9-10 am.
2. Recall there is a quiz Monday February 7 in the first forty minutes of class, covering the first four lectures. There is a sample quiz posted.
3. Reading: Note that we'll be doing some geometry that isn't in Miller directly; it may be helpful to consult Sections 1.3-5 of Kolman and Beck. Subsequent to that, we'll start the beginning of Chapter 6 in Miller (or very similarly Chapter 2 of Kolman and Beck.)
4. Recall from class that a rectangle R in \mathbb{R}^n is a set

$$R = \{(x_1, \dots, x_n) : a_i \leq x_i \leq b_i\}$$

where $a_i \leq b_i$ are real numbers for each $i = 1, \dots, n$. Write down an algebraic (meaning that your proof should involve equations, not a picture) proof that rectangles are convex.

5. Prove that if S is a convex subset of \mathbb{R}^n and $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a linear function, then the image $f(S)$ is a convex subset of \mathbb{R}^m . [Recall that a linear function f has the properties that $f(\mathbf{x} + \mathbf{y}) = f(\mathbf{x}) + f(\mathbf{y})$ and that $f(c\mathbf{x}) = cf(\mathbf{x})$ for all $c \in \mathbb{R}$.]
6. Draw the set of convex combinations of $(1, 3)$, $(2, 1)$, $(3, 4)$, and $(6, 2)$ in \mathbb{R}^2 . Give two different ways of writing $(2, 2)$ as a convex combination of these points. What are the extreme points of the set you found?
7. Draw the set of convex combinations of $(1, 6)$, $(2, 1)$, $(3, 2)$ and $(6, 1)$ in \mathbb{R}^2 . What are the extreme points of the set you found?
8. Sketch the set of feasible solutions to the constraints

$$\begin{cases} 6x_1 + 4x_2 + 9x_3 \leq 36 \\ 2x_1 + 5x_2 + 4x_3 \leq 20 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

and determine what the extreme points of the set are. [Your sketch quality will not be graded harshly.]