MATH 354: Homework 2

Due: February 3, 2022 at 11:00 am

1. Upcoming office hours are Monday January 31 3-4 pm, and Thursday February 3 9-10 am.
2. Read Sections 4.3-9 in Miller. [It may also be helpful to consult Sections 1.1 and 1.2 of Kolman and Beck.]
3. Reminder that there is a quiz coming up on February 7 in the first thirty minutes of class. It will cover the material of the first four lectures, that is, through January 31st. A sample quiz and solutions will be posted sometime early next week.

For problems 4-6, put the linear programming problem in (a) standard form, both as equations and in matrix notation, and (b) canonical form, both as equations and matrix notation. For 4 and 5, feel free to start with the equations given in the homework solutions.

4. Problem 3 from Assignment 1. For this problem, also answer the following: What are the values of the slack variables at the optimal solution you found last week? What do those numbers mean in terms of the original problem?
5. Problem 4 from Assignment 1. For this problem, also answer the following: What are the values of the slack variables at the optimal solution you found last week? What do those numbers mean in terms of the original problem?
6. Maximize $z = 2x_1 + 3x_2 + 5x_3$ subject to the constraints

$$\begin{cases} 4x_1 - x_2 \geq 3 \\ x_1 + x_2 + x_3 \leq 5x_1 \\ x_3 = 7 \\ x_1 \geq 0, x_3 \geq 0 \end{cases}$$

For the remaining problems, actually answer the question posed.

7. Write down a linear programming problem in standard form that has no feasible solutions (along with a justification of why it doesn't).
8. Consider a linear programming problem in canonical form, written "maximixe $\mathbf{c}^T\mathbf{x}$ subject to the constraints $\mathbf{A}\mathbf{x} = \mathbf{b}$ and $\mathbf{x} \geq \mathbf{0}$." Let r, s be two nonnegative real numbers such that $r + s = 1$. Prove that if $\mathbf{x}$ and $\mathbf{y}$ are feasible solutions, so is $r\mathbf{x} + s\mathbf{y}$.