MATH 354: Homework 13

Due: Monday May 2, 2022 at 11:00 am. Note unusual date!

1. Remaining office hours are Monday April 25 3-4 pm, Thursday April 28 9-10 am, Monday May 2 9-10 am, Wednesday May 4 4-5 pm.

2. Recall that the final is Thursday May 5 8-11 am in the usual room. The sample final will be posted by Monday May 2 at the latest. The TA review session is Wednesday May 4 1-2 pm; there will subsequently be TA office hours 2-3 pm. As usual the review session will be recorded and posted online promptly.

3. Graded finals will be available to be viewed Friday May 6 9-10:30 am and Monday May 9 10 am-12 pm at my office, which is Hill Center 515 on Busch Campus.

4. Reading is Sections 5.3-5 in Kolman and Beck.

5. Solve the transportation problem with

\[
C = \begin{bmatrix}
5 & 5 & 7 & 2 \\
4 & 8 & 4 & 2 \\
6 & 2 & 3 & 1 \\
9 & 5 & 4 & 3
\end{bmatrix}, \quad s = \begin{bmatrix}
50 \\
80 \\
40 \\
40
\end{bmatrix}, \quad d = \begin{bmatrix}
75 \\
50 \\
60 \\
25
\end{bmatrix}.
\]

6. The MG Auto Company has three factories in Los Angeles, Detroit, and New Orleans, and two major distribution centers in Denver and Miami. The plants can produce 1000, 1300, and 1200 cars per week respectively. The distribution centers have a demand of 2300 and 1400 cars each week respectively. There is a penalty in contract between the auto company and the distribution centers of $200 for each car less than the demand received by Denver and $300 for each car less than the demand received by Miami. The shipping costs for one car between the factories and distribution centers are as follows.

<table>
<thead>
<tr>
<th></th>
<th>Denver</th>
<th>Miami</th>
</tr>
</thead>
<tbody>
<tr>
<td>Los Angeles</td>
<td>$80</td>
<td>$215</td>
</tr>
<tr>
<td>Detroit</td>
<td>$100</td>
<td>$108</td>
</tr>
<tr>
<td>New Orleans</td>
<td>$102</td>
<td>$68</td>
</tr>
</tbody>
</table>

What shipping plan results in the smallest total money spent on shipping costs and contract penalties? (Note that the first step here is to decide how to represent this as a balanced problem, which is meant to require minor thinking.)

7. Find the maximal flow from the source \( s \) to the sink \( t \) through the network at the top of the next page. Then locate a minimal cut.
8. A water pipeline company provides fresh water in a given area. The diagram below shows the pipeline network connecting an origin $O$ to a town $T$.

(a) What is the maximal flow of water from the origin to the town?
(b) Due to population growth, the pipeline company needs to increase the volume of water it provides to the town by two units. The cost of expanding a link is proportional to the current link capacity; for example, it costs $500n$ to expand a link with current capacity 5 to capacity $5 + n$. Which pipelines should be expanded and by how much for the cheapest expansion?

9. A campus advising center has 149 students who would like an advising appointment this week. The advisers present have 150 slots on their total schedules. The students have the following majors.

<table>
<thead>
<tr>
<th>Biology</th>
<th>Chemistry</th>
<th>Engineering</th>
<th>Computer Science</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>40</td>
<td>34</td>
<td>39</td>
</tr>
</tbody>
</table>

The advisers present have various kinds of expertise. Adviser A has 30 appointments available and can help Computer Science and Engineering majors. Adviser B has 40 appointments available and can help Biology and Chemistry majors. Adviser C has 35 appointments available and can help Engineering and Chemistry majors. Adviser D has 45 appointments available and can help Computer Science and Engineering majors. Decide how to model this as a network, and determine the maximum number of students who can get an advising appointment this week.