

MATH 354: Homework 1

Due: January 27, 2022 at 11:00 am

1. Upcoming office hours are Thursday January 20 3-4 pm, Monday January 24 3-4 pm, and Thursday January 27 9-10 am.
2. Read Sections 3.4-5, 4.3-9 in Miller. [Or equivalently the same material in Section 1.1 of Kolman and Beck.]

For each of the problems below, (a) Define the variables in the problem; (b) Write down the objective function and the constraints; (c) Draw the set of feasible solutions in the plane; (d) Use the method shown in class to find the optimal solution to the problem in the real numbers.

3. A coffeeshop owner has on hand 120 kg of beans of coffee B and 160 kg of coffee C, and wishes to blend them into two coffee blends, Super and Deluxe. Each kilogram of the Super blend contains .5 kg of coffee B and .5 kg of coffee C, whereas each kilogram of Deluxe contains .25 kg of Coffee B and .75 kg of Coffee C. If the profit on each kilogram of Super is 20 cents and the profit on each kilogram of Deluxe is 30 cents, how should the coffee be packed to maximize profit?
4. A container manufacturer is considering the purchase of two different types of cardboard-folding machines, Model A and Model B. Model A can fold 40 boxes per minute and requires 1 attendant, whereas Model B can fold 50 boxes per minute and requires 2 attendants. Suppose the manufacturer must fold at least 360 boxes per minute and cannot afford to have more than 12 employees for the folding operation. A Model A machine costs \$15,000 and a Model B machine costs \$20,000. What number of Model A machines and model B machines should the manufacturer buy to minimize cost?
5. Sketch the set of feasible solutions satisfying the constraints

$$\begin{cases} x + y & \leq 5 \\ 2x + y & \leq 8 \\ x, y & \geq 0 \end{cases}$$

What is the maximum value of $x + 3y$ subject to these constraints? What about $5x + 3y$?

For each of the problems below, (a) Define the variables in the problem, (b) Write down the objective function and the set of constraints.

6. A book publisher is planning to bind its latest offering in three different bindings: paperback, book club, and library. Each book goes through a sewing and gluing process. Paperbacks take 2 minutes to sew and 4 to glue, book club books take 2 minutes to sew and 6 to glue, library books take 3 minutes to sew and 10 to glue. The sewing process is available for 7 hours a day and the gluing process is available for 10 hours per day. Assume profits are \$.50 on a paperback edition, \$.80 on a book club edition, and \$1.20 on a library edition. How many books should be produced in each binding to ensure profits are maximized?

7. The adviser of an investment fund has up to \$200,000 to invest in a utilities stock which pays a 9% dividend per year, an electronics stock which pays a 4% dividend per year, and a bond paying 5% interest per year. Suppose that the amount invested in stocks cannot be more than half of the total money invested, the amount invested in the utilities stock cannot exceed \$40,000, and the total amount in the bond must be at least \$70,000. What policy should be pursued to maximize return?