Math 354, Section 04
Linear Optimization

Sample Final

Instructions: You have three hours to complete the exam. There are six questions, worth a total of 60 points. Partial credit will be given for progress toward correct solutions where relevant. You may not use any books, notes, calculators, or other electronic devices.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

1. Consider the following linear programming problem: Maximize $z = 3x_1 + 2x_2 - x_3 + 3x_4$ subject to the constraints

$$\begin{cases} 2x_1 + x_2 + 2x_3 \leq 16 \\ 3x_1 + x_2 + x_4 \leq 30 \\ 5x_1 + x_2 + 3x_3 + 2x_4 \leq 35 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

The final tableau for the simplex method applied to this problem is shown below.

	x_1	x_2	x_3	x_4	u_1	u_2	u_3	z	
x_2	2	1	2	0	1	0	0	0	16
u_2	$-1/2$	0	$-5/2$	0	$-1/2$	1	$-1/2$	0	$9/2$
x_4	$3/2$	0	$1/2$	1	$-1/2$	0	$1/2$	0	$19/2$
	$11/2$	0	$13/2$	0	$1/2$	0	$3/2$	1	$121/2$

(a) [1pts.] What basic feasible solution does this tableau represent?

(b) [2pts.] What is the dual problem to the linear programming problem above, and what is its optimal solution?

(c) [7pts.] What is the best solution to this problem if we add the constraint that x_1, x_2, x_3, x_4 are integers?

This page is additional space for your solution to Problem 1.

2. Rutgers landscaping is purchasing plants for campus, and needs 350 plants with red flowers and 300 plants with white flowers. A flat of poppies contains 15 plants with red flowers and 20 plants with white flowers and costs $50. A flat of geraniums contains 20 plants with red flowers and 10 plants with white flowers and costs $40. A flat of petunias contains 15 plants with red flowers and 5 plants with white flowers and costs $35.

   (a) [6pts.] What is the cheapest combination of flats of plants that landscaping can purchase to meet the campus's needs? [Hint: The simplex method is not the quickest way to analyze this.]

   (b) [4pts.] Because of campus expansion, the university expects that next year the number of plants with white flowers needed will be 500. What should the university's buying plan be next year? [Hint: Ditto.]

This page is additional space for your solution to Problem 2.

3. [10pts.] An furniture company needs to ship office chairs from its four factories F_1, F_2, F_3, F_4 to its three distribution sites D_1, D_2, D_3. Below, the vector $\mathbf{s}$ gives the supply at each factory, the vector $\mathbf{d}$ give this demand at each distribution site, and the matrix C gives the costs of shipping one unit between each pair of locations. Determine the cheapest shipping plan and its cost.

$$\mathbf{s} = \begin{bmatrix} 100 \\ 60 \\ 90 \end{bmatrix}, \qquad \mathbf{d} = \begin{bmatrix} 60 \\ 80 \\ 70 \\ 40 \end{bmatrix}, \qquad C = \begin{bmatrix} 6 & 4 & 3 & 5 \\ 7 & 4 & 8 & 6 \\ 8 & 3 & 2 & 5 \end{bmatrix}.$$

This page is additional space for your solution to Problem 3.

4. (a) [5pts.] A courier service has five bicycle couriers $A_1, A_2, \ldots, A_5$ at various places in New Brunswick and five packages $P_1, \ldots P_5$ to be picked up. The number of minutes it will take the ith courier A_i to reach the pickup location for the package P_j is given as the entry c_{ij} of the matrix below. What assignment of couriers to packages minimizes the travel time?

$$\begin{bmatrix} 3 & 2 & 7 & 4 & 8 \\ 5 & 4 & 3 & 8 & 5 \\ 2 & 7 & 9 & 1 & 2 \\ 4 & 2 & 6 & 5 & 7 \\ 3 & 8 & 4 & 6 & 6 \end{bmatrix}$$

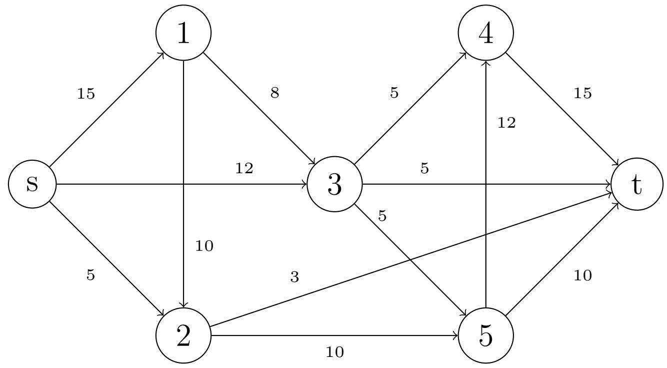
(b) [5pts.] Six students $L_1, \ldots, L_6$ from a linear programming class plan to partner with six students $C_1, \ldots, C_6$ from a chemistry class to do final projects studying chemical reactions. Students would of course like to partner with their friends. The matrix of who is friends with whom is shown below; a 1 in the ij entry indicates that student L_i and C_j are friends and a 0 that they are not.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Determine the maximum number of partnerships that can be between two friends.

This page is additional space for your solution to Problem 4.

5. Consider the following pipeline network.



(a) [5pts.] Determine the maximum flow through this network.

(b) [3pts.] Locate a minimum cut in this network.

(c) [2pts.] You can choose one pipeline to expand in order to increase the flow through the network. Which pipeline do you expand, and by how much can you usefully expand it?

This page is additional space for your solution to Problem 5.

6. Consider the following linear programming problem: maximize $z = 3x_1 + x_2 + 2x_3$ subject to

$$\begin{cases} x_1 - 3x_2 + 3x_3 \leq 30 \\ x_1 + x_2 + x_3 = 20 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

   (a) [5pts.] Solve this problem using the simplex method.

   (b) [2pts.] Suppose the objective function above is replaced with $z = 3x_1 + c_2x_2 + 3x_3$. For what values of c_2 is the solution you found still optimal?

   (c) [3pts.] Suppose the first constraint above is replaced with $x_1 - 3x_2 + 3x_3 \leq 10$. What is the new optimal solution?

This page is additional space for your solution to Problem 6.

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