Math 311H
Honors Introduction to Real Analysis

Quiz

Instructions: You have 30 minutes to complete the quiz. There are three questions, worth a total of fifteen points. Partial credit will be given for progress toward correct solutions where relevant. You may not use any books, notes, calculators, or other electronic devices.

Name: _____

RUID: _____

Question	Points	Score
1	5	
2	5	
3	5	
Total:	15	

1. For $n \in \mathbb{N}$, let P_n be the statement "$n^2 + 5n + 1$ is an even number".

   (a) [3pts.] Prove that if P_n is true, then P_{n+1} is true.

   (b) [1pts.] For which n is P_n true?

   (c) [1pts.] How do you reconcile parts (a) and (b)?

2. (a) [4pts.] Let A and B be two nonempty bounded subsets of $\mathbb{R}$. Prove the equality $\sup(A \cup B) = \max\{\sup A, \sup B\}$.

   (b) [1pts.] Give an example to show that it is not necessarily true that $\sup(A \cap B) = \min\{\sup A, \sup B\}$. Note that your example should have nonempty intersection.

3. [5pts.] Recall that the product of two sets A and B is $A \times B = \{(a, b) : a \in A, b \in B\}$. Prove that the product of two countable sets is countable.