Math 311H
Honors Introduction to Real Analysis

Sample Midterm 2

Instructions: You have 80 minutes to complete the exam. There are six questions, worth a total of thirty points. Partial credit will be given for progress toward correct solutions where relevant. You may not use any books, notes, calculators, or other electronic devices.

Name: ______________________________

Question	Points	Score
1	5	
2	5	
3	4	
4	5	
5	6	
6	5	
Total:	30	

1. For each of the following things, either give an example of the described object (no need to justify it) or write a sentence saying why this is impossible.
   (a) [1pts.] A perfect set with exactly three limit points.
   (b) [1pts.] A set E with $E^\circ = \emptyset$ and $\overline{E} = \mathbb{R}$.
   (c) [1pts.] A connected set consisting of only irrational numbers.
   (d) [1pts.] A noncompact set A and an open cover of A which has a finite subcover.
   (e) [1pts.] A continuous surjective function from the Cantor set to the interval $[0, 1]$.

2. [5pts.] Let $f\colon \mathbb{R} \to \mathbb{R}$ be a continuous function and $E \subseteq R$. Prove that $f(\overline{E}) \subseteq \overline{f(E)}$.

3. [4pts.] Suppose that f, g are two functions with the same domain A such that $f(x) \leq g(x)$ for all $x \in A$, and say that $\lim_{x\to c} f(x) = L_1$ and $\lim_{x\to c} L_2$ both exist for some limit point c of A. Prove that $L_1 \leq L_2$.

4. [5pts.] Let f be uniformly continuous on a bounded set A. Prove that the image $f(A)$ is also bounded.

5. For each of the following pairs of sets, either give an example of a continuous function $f\colon A \to \mathbb{R}$ whose image is $f(A) = B$ (no need to justify your answer) or explain why no such function exists.

   (a) [2pts.] $A = [0, 1]$; $B = [1, 2)$

   (b) [2pts.] $A = (0, 1]$; $B = [1, \infty)$

   (c) [2pts.] $A = (0, 1)$; $B = (0, 1) \cup (3, 4)$

6. A map $f\colon \mathbb{R} \to \mathbb{R}$ is called *open* if for every $O \subset \mathbb{R}$ an open set, the image $f(O)$ is also open.

   (a) [1pts.] Give an example of a continuous function $f\colon \mathbb{R} \to \mathbb{R}$ which is not open, including an open set O such that $f(O)$ is not open.

   (b) [4pts.] Prove that any open continuous map f is strictly monotone. [Hint: For any $a < b$ in $\mathbb{R}$, where must the minimum and maximum values of f on $[a, b]$ lie?]